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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This booklet presents a unit on sets. The topics covered are set membership, conservation of sets, the empty set, 1-to-1 correspondence, matching sets, and the concept of subsets. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

MINNE

Math And Science Teaching project

UNIT II

SETS

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

Unit II

Sets

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We are deeply indebted to the many teachers who
used earlier versions of this material
and provided suggestions for
this revision

CONTENTS

* Starring indicates content which is particularly important to the sequential development or evaluation of the program. We ask that all participating teachers try this starred material. It is expected that much of the remaining material will also be used; how much will depend on individual class needs and time available.

Teacher Background on Sets

Teaching Objectives

1. Foster understanding of the meaning of set.
2. Have the children begin to use the word set naturally and appropriately. Incorrect usage of the term set should be overlooked at this stage.
3. Have the children learn the ways of describing a set; be able to recognize and determine the membership of a set from its description.
4. Establish that rearrangement of the members does not change the members of a set.
5. Stimulate a beginning awareness of the concept of the empty set.

Why We Teach Children About Sets

The concept of set as taught in the elementary school has simply borrowed from advanced mathematics the vocabulary and basic operations of sets. This is part of the current aim to introduce children as early as possible to the major concepts common to all mathematics.

The concept of set permeates all mathematics from the kindergarten through the graduate school. Modern mathematics places great emphasis on precision in language and exactness of expression. Since the language and operations of sets are precise and exact it is only natural that the idea of sets be introduced as early as possible in the child's mathematical development.

Through the use of elementary set terminology and operations, we are able to describe mathematical ideas and perform operations that would prove difficult and cumbersome in traditional terms.

When Cantor, in the 1880's, introduced the concept of set, mathematicians immediately recognized that this concept is basic to the idea of number. It helps us understand the meaning of numbers and of the operations, such as addition and multiplication, which we perform on numbers. Like many other simple ideas, it took a long time for Man to discover the concept of set. Now that we know it, we find it simple

enough to teach to younger children. If we do this, children find arithmetic easier to learn.

When children come to school they have a very inadequate concept of number. It is felt that the concept of set is simpler and can be approached more concretely than the concept of number. Number is then introduced as a property of a set.

What Set Means

A set, as the term is used by mathematicians, may:

a) be composed of an infinite number of members, as the set of all odd numbers.

b) be composed of a finite number of members, such as the set of all children of school age in the United States (this number can be obtained).

c) be the set of all men now Secretary-General of the United Nations, which is a set composed of one member.

d) be the set of all living persons who have seen King Solomon in person, which would be the empty set. The number of members in the empty set is zero.

The members of a set may be similar, as in a set of apples, or dissimilar, as in the set of all things in a lady's purse. They may be concrete objects, as in the above examples, or abstract, as in the set of ideas such as beauty, intelligence, courtesy. The members of a set may themselves be sets.

Being a set is not a property of reality, like being red. It is a subjective way of organizing the world of reality (and unreality, too!) To the auto parts salesman, a car is essentially a set of parts. To the automobile manufacturer, the car is more than just the same set of parts—it is a commodity with unique selling potential.

The concept of set is usually left undefined by mathematicians. When the concept is defined, it is generally done implicitly, by saying when sets are equal. Sets are equal (or identical, or the same set) if, and only if, they have the same members. Synonyms for set are collection, class.

The best ways for children to learn about sets are

1. by hearing the term used appropriately
2. by considering sets of concrete objects
3. by learning what operations (taking away, adding, or substituting a member) change a set
4. by discovering the operations (such as rearranging the members or their names) that leave sets unchanged.

Conservation of Sets

Research has shown that many children do not realize that rearranging the members of a set leaves the number property unchanged. It is easier for children to come to this realization if they first understand that rearranging the members of a set leaves the set unchanged.

The order of the members of a set, either in space or description, is irrelevant. The purpose of the story "David's Promise" and the activities following it, is to demonstrate that rearranging the members of a set does not change the set. This is referred to as "conservation of sets under rearrangement of the members". Similarly, in naming or describing a set by listing its members, the order in which the members are listed is unimportant.

Describing Sets

One way of identifying or describing sets is to list the members. They might be listed by using pictures

or by using names

{JFK, Abe Lincoln, Ugboo}

Another method is to state the conditions that members of the set satisfy and nothing else satisfies. In other words, the membership requirements or limits or bounds of the set are stated. An example is "the set of all Presidents of the United States in 1962".

For some sets, giving conditions is more convenient than listing. It would be possible to list all of the members of a class in a particular school, but this obviously would be impractical. Such a relatively large set would be much simpler to describe by giving conditions "the set of all members of class A in school B" .

If, on the other hand, the set to be identified was "the set of all whole numbers from 1 through 5", the set could be conveniently listed as follows:

1, 2, 3, 4, 5

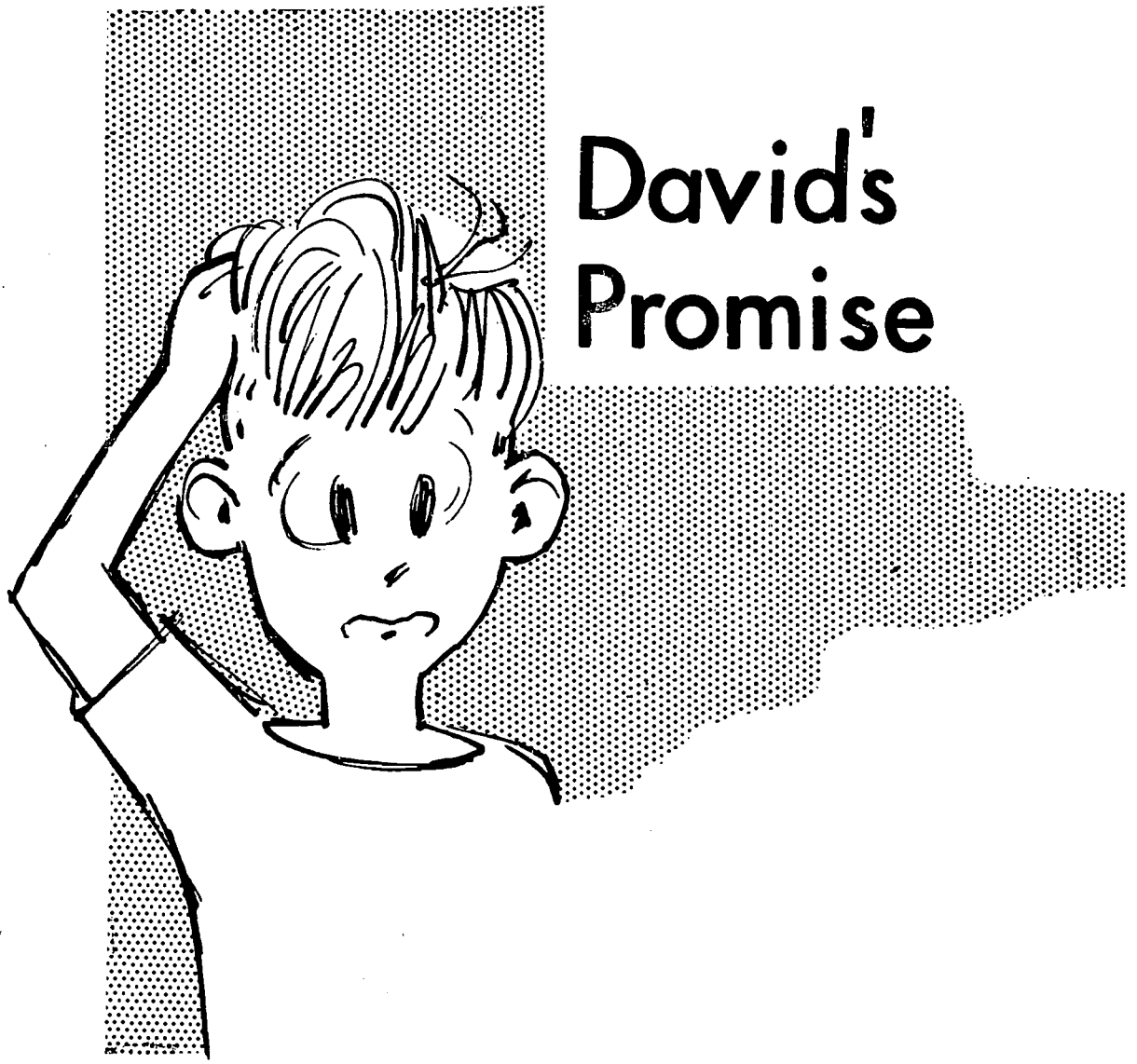
Any description, whether a long phrase or a list, can serve as a name for a set. "The set of all whole numbers from 1 through 5" and {1, 2, 3, 4, 5} are two different names for the same set.

In writing the name of a set, curly braces are used as abbreviations for "the set whose members are" or "the set of all", as in {whole numbers from 1 through}.

Note: Examples involving numbers are for the teacher's reference only and should not be used with the children. The concept of number is not introduced until Unit IV.

A description does not define a set unless it is possible to tell from the description (and the context) whether or not something is a member. The phrase "the set of all fruit in the room" does not define a set in a room which contains artificial as well as real fruit. The phrase "the set whose members are an apple, an orange, and a lemon" does not define a set, unless the context determines which apple, orange, and lemon.

The story "David's Promise" introduces the term set and indicates that a set is unchanged after rearranging its members.

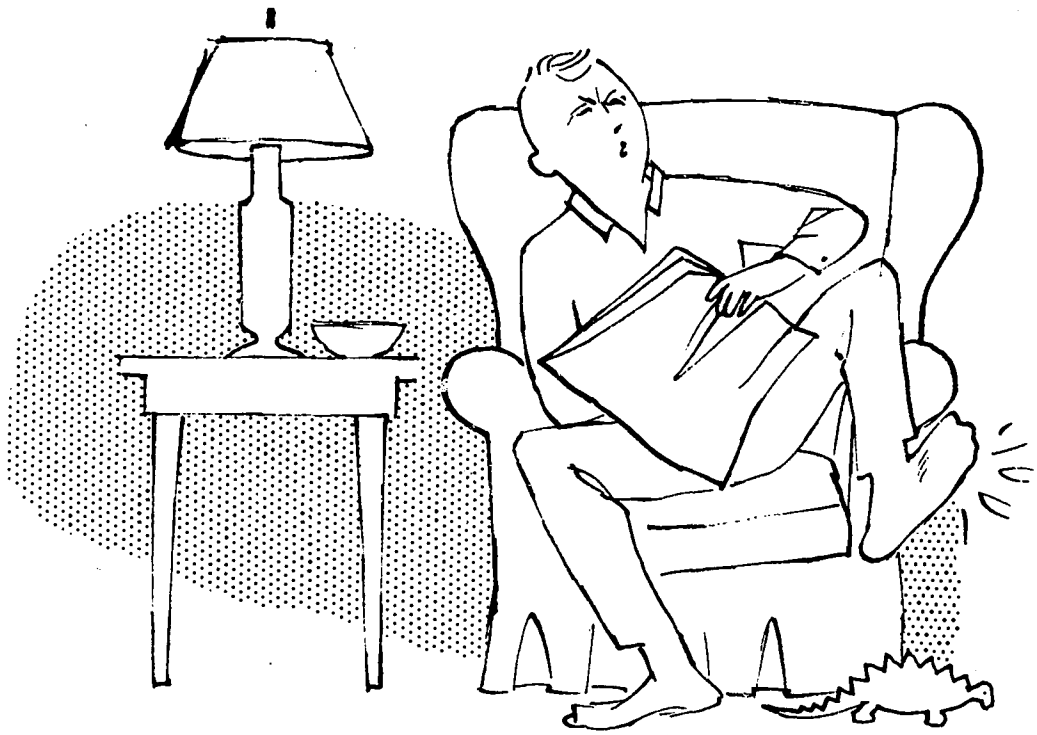


David's Promise

Conservation of Sets
Under Rearrangement
of the Members

School is over for the day. As David walks into the house he is whistling a happy tune, but the whistle fades away as he hears his father's voice cry, "Ouch! Spikes! Who left this toy dinosaur next to my chair? I just stepped on those sharp points!"

Then he hears his big sister Kathy's voice cry, "Cars, cars, cars, all over the table! I don't have a place to put my books!"



"Ouch! Spikes! Who left this toy dinosaur next to my chair?"

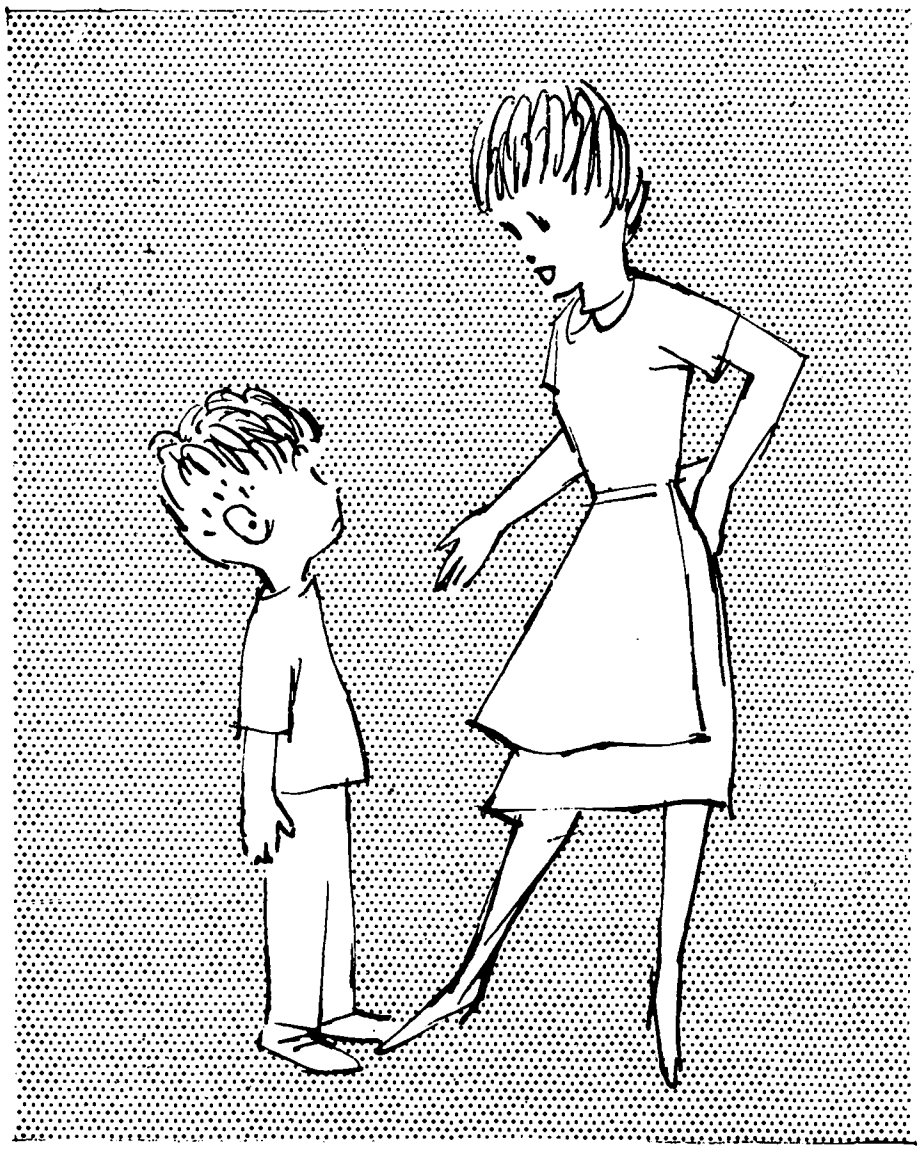
"Cars, cars, cars, all over the table!"

His mother sighs, "David, David, you must learn to keep your toys in their proper places."

"Oh, dear, complaints, complaints," thinks David. But then he sees that special look on his mother's face as she says, looking straight at him, "Remember your promise!"

David feels a little ashamed. "Oh, Mom," he begins slowly, "I really tried for a while but I guess I forgot again. "I'll start all over right now, honest I will!"

"All right, David," his mother smiles. "I think you really do try. See what a good job you can do of straightening up your toys while I go to the dentist."



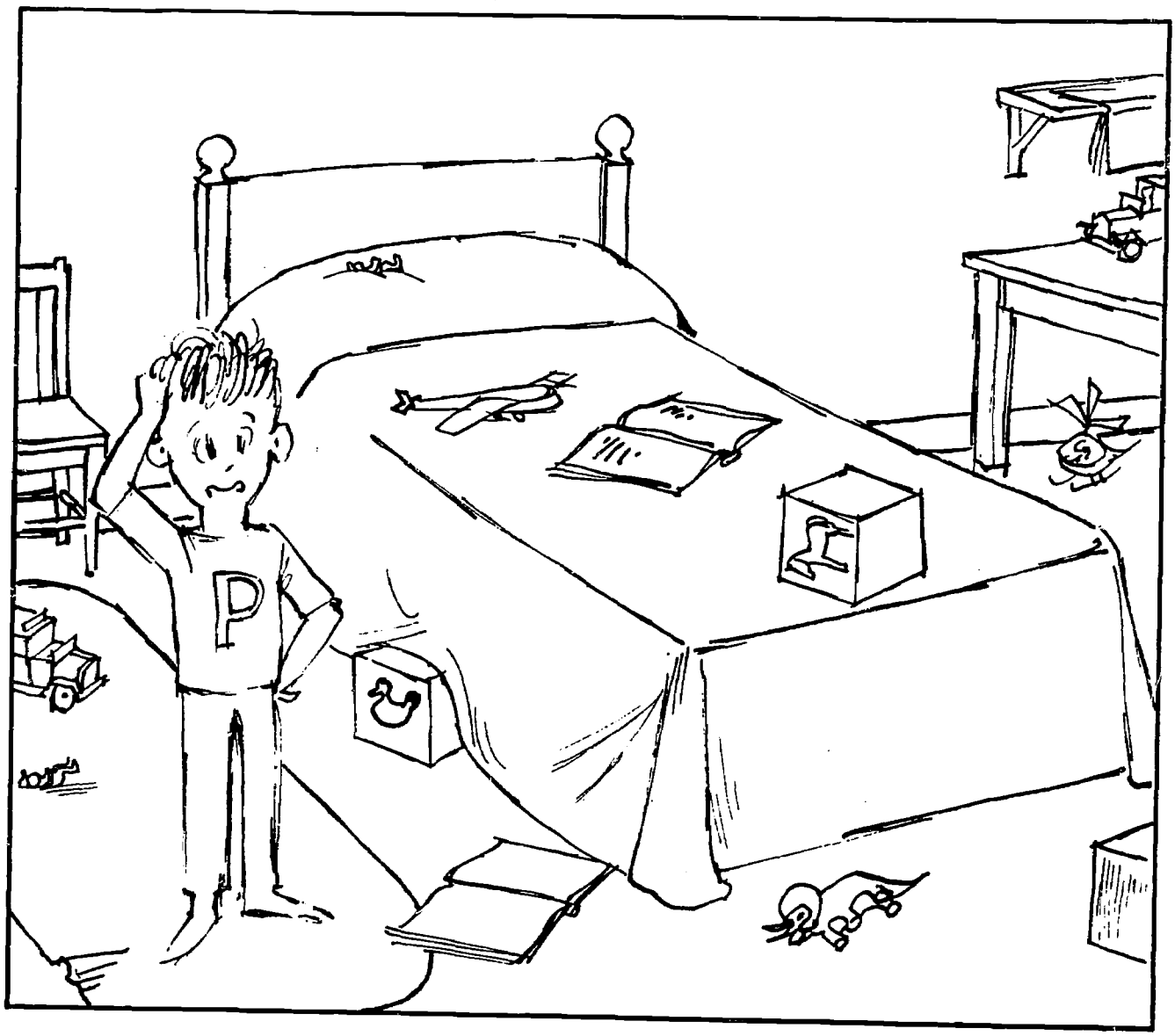
"See what a good job you can do of straightening up your toys while I go to the dentist."

David picks up the toy dinosaur from the floor and collects the cars from the table. He walks into his room and looks around.

His toys really are scattered all over, and he scratches his head while trying to decide where to begin. Then suddenly his mouth widens into a happy grin.

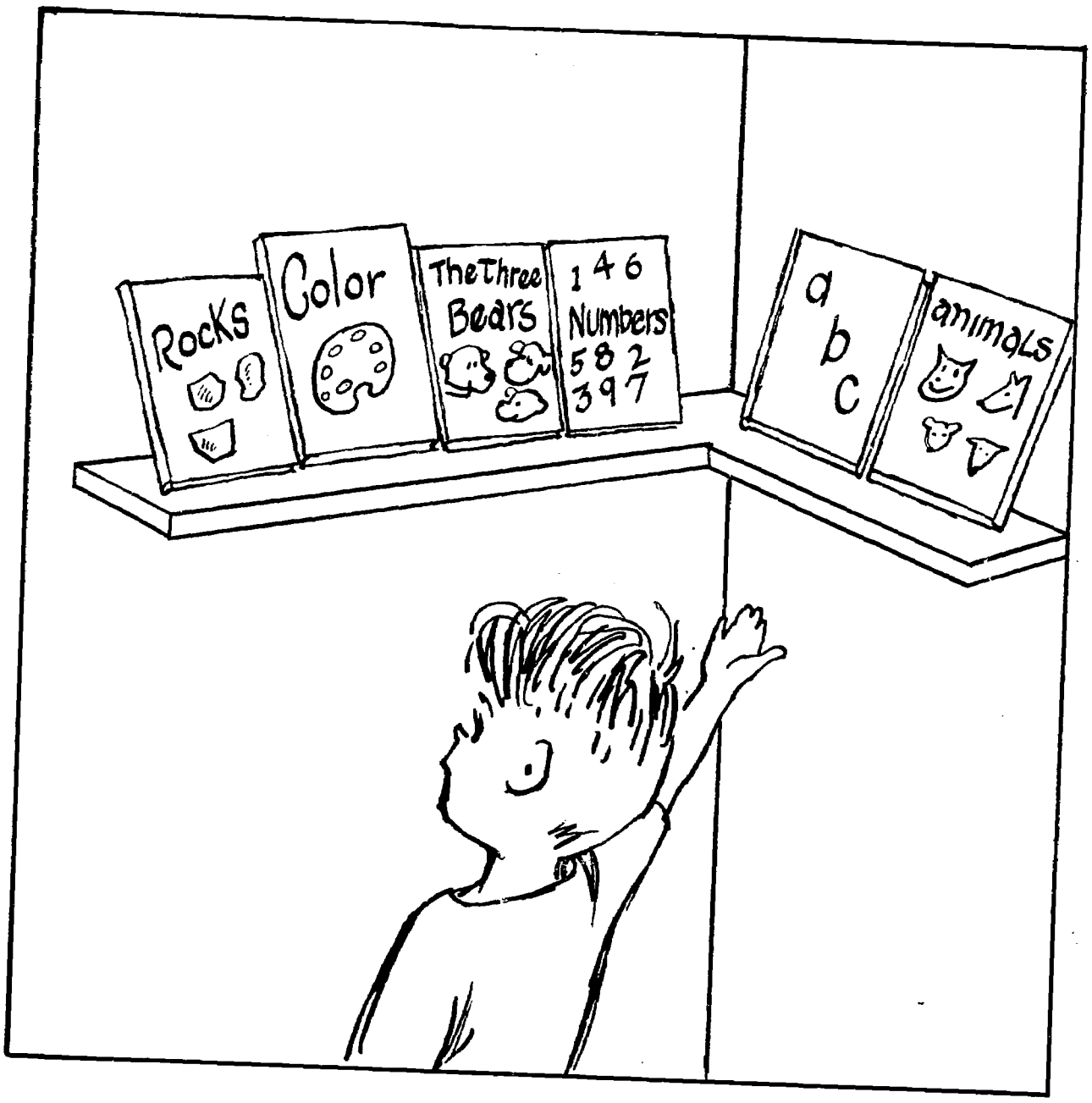
"I know, I'll really surprise Mother! I'll get all my toys together and arrange them in sets just the way Mom puts the set of dishes on the shelf. This will make her really happy."

So David scrambles upstairs and downstairs. He finds old toys and new ones, games and cars, books and blocks and brings them all to his room.



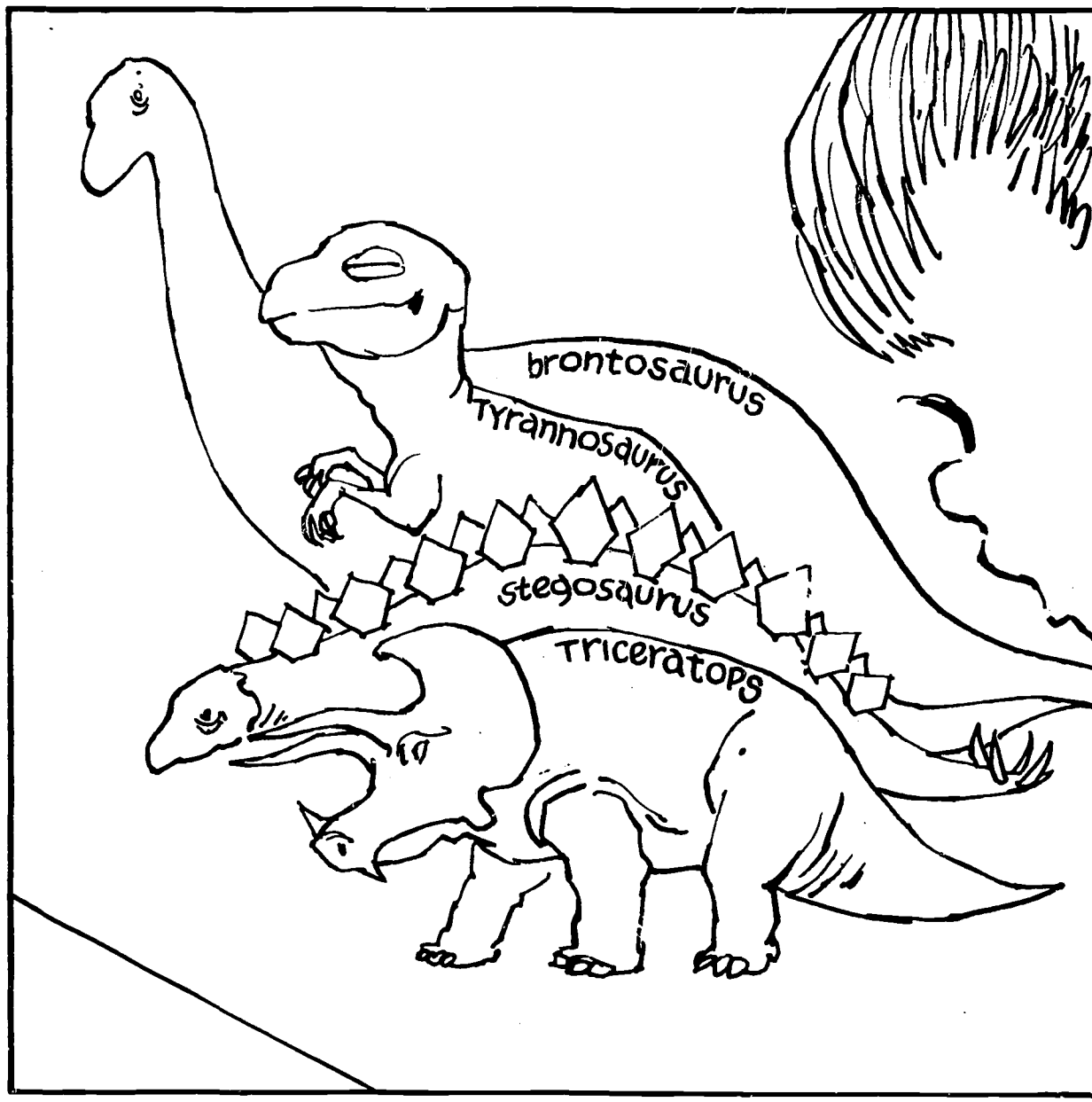
He scratches his head while trying to decide where to begin.

After David finds his favorite books he thinks to himself, "My bookshelf is a good place for this set. I'll put all of the books together. Here's my Rock book, my Three Bears book, my Numbers book, and my ABC book."



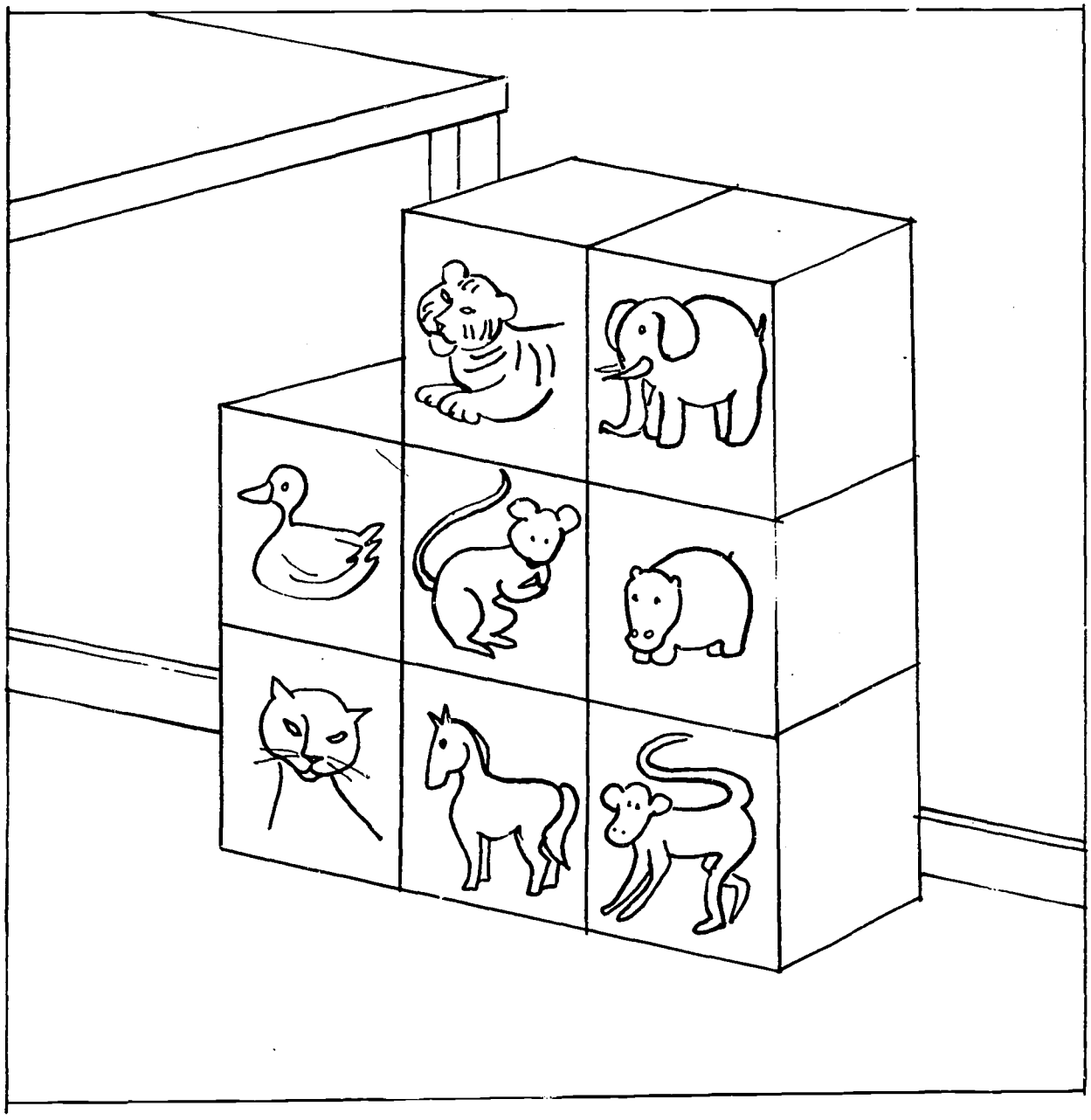
"My bookshelf is a good place for this set."

"What's that peeping out from under the chair? It looks like Mr. Brontosaurus (Bron - to - sa - rus), my largest dinosaur. I know what I'll do with him. I'll pretend this part of the table is a museum and then I can put the set of dinosaurs close to each other here. Come on Stegosaurus (Steg - o - sa - rus), you, Tyrannosaurus (Ti - ran - o - sa - rus), and funny Triceratops (Tri - ser - a - tops). I have a museum for you to live in now."



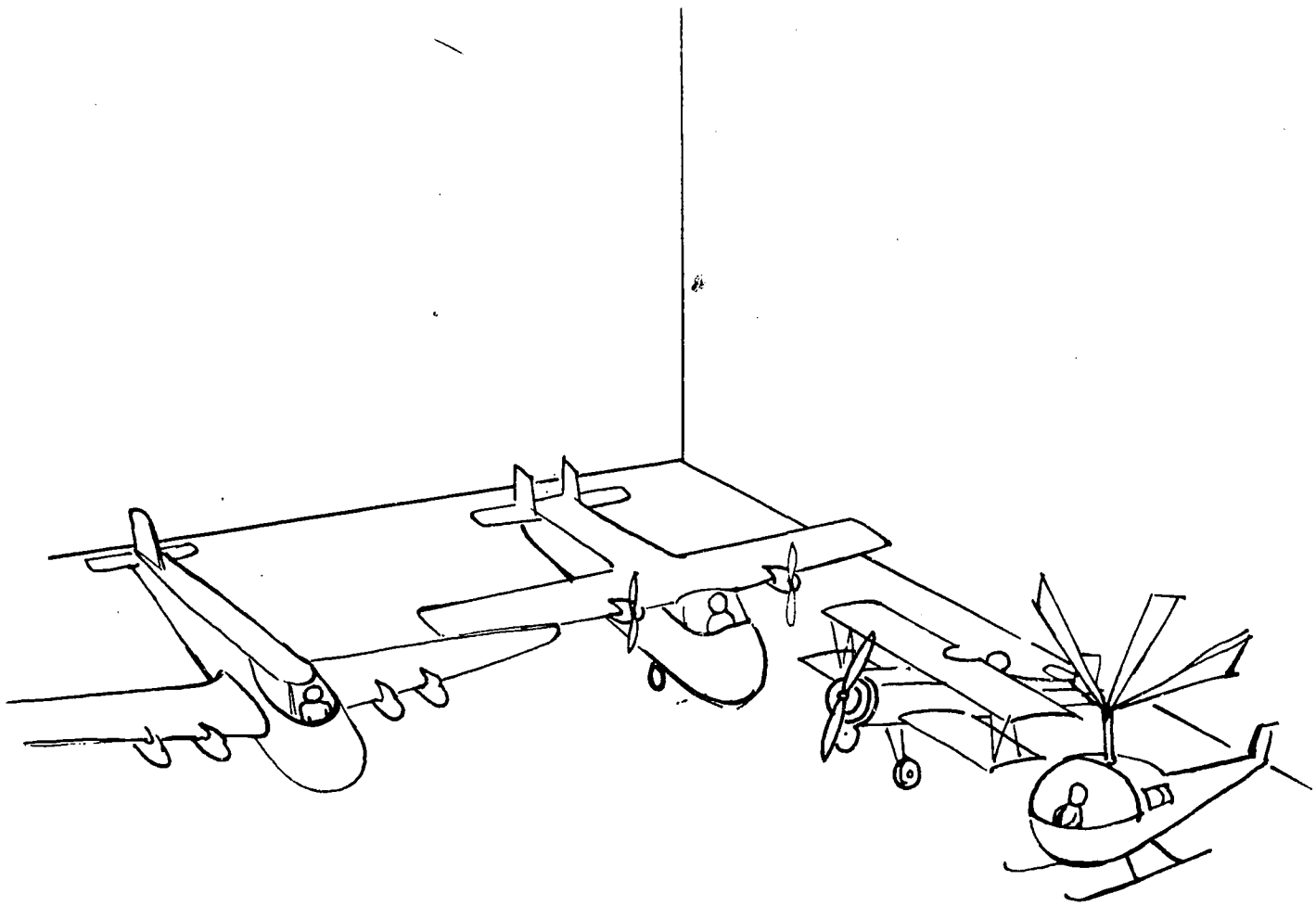
"I'll pretend this half of the table is a museum and then I can put the set of dinosaurs close to each other here."

"My set of blocks is the easiest set to put together. I'll just pile them up here by the window."



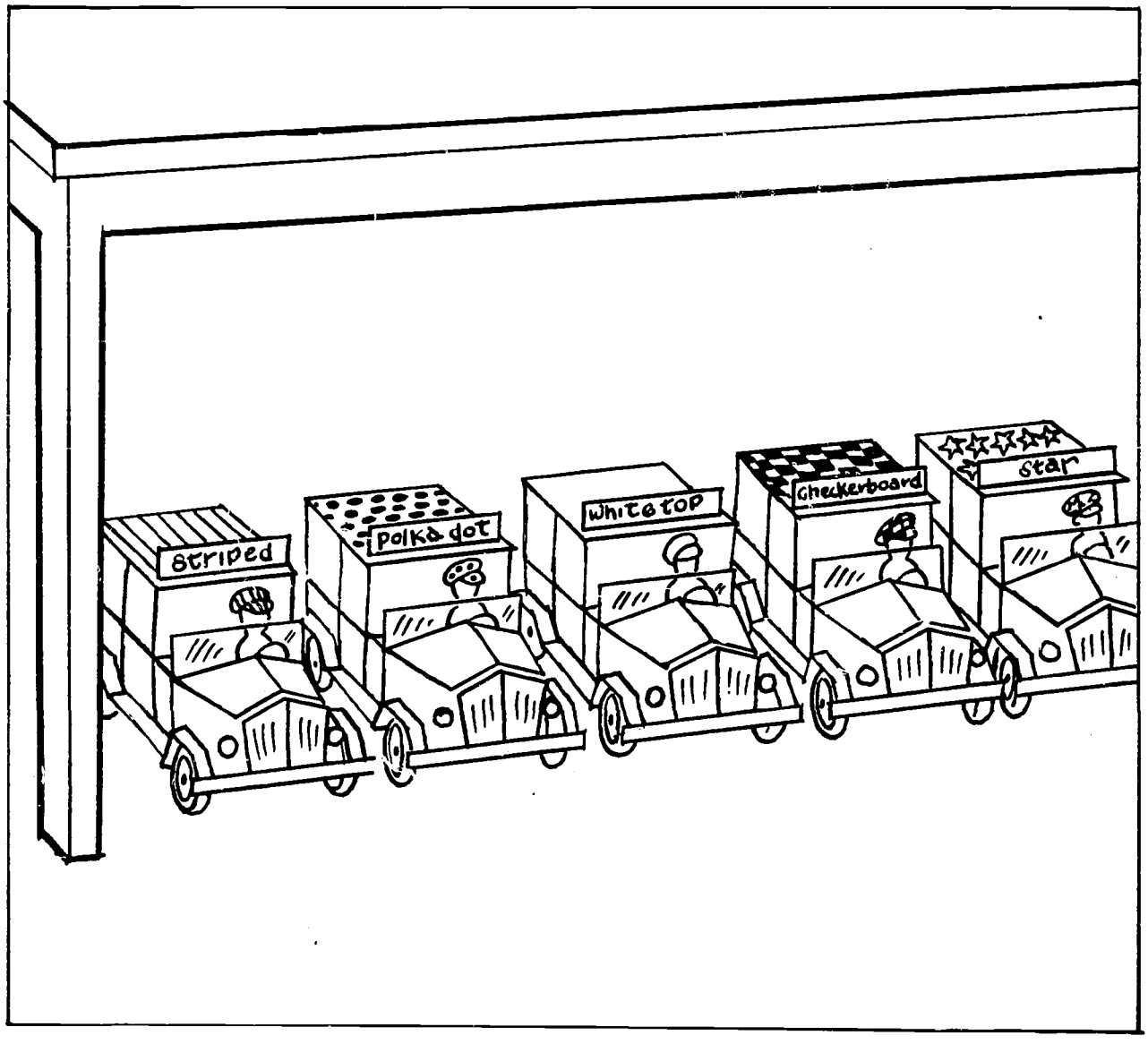
"My set of blocks is the easiest set to put together."

David looks around the room for a good place for his set of planes. "The other side of the table will be my hangar," he finally decides, "and I will pretend that each of the pilots has taxied his plane into the hangar. Now my set of planes is right next to the set of dinosaurs.



"Now my set of planes is just next to my set of dinosaurs."

"But where shall I put my toy taxis and their drivers? What can I pretend is a garage? I guess here under the table will do. Have I got them all? Here's the Striped Cab, the Polka Dot Cab, the White Top, the Checkerboard and the Star Cab."

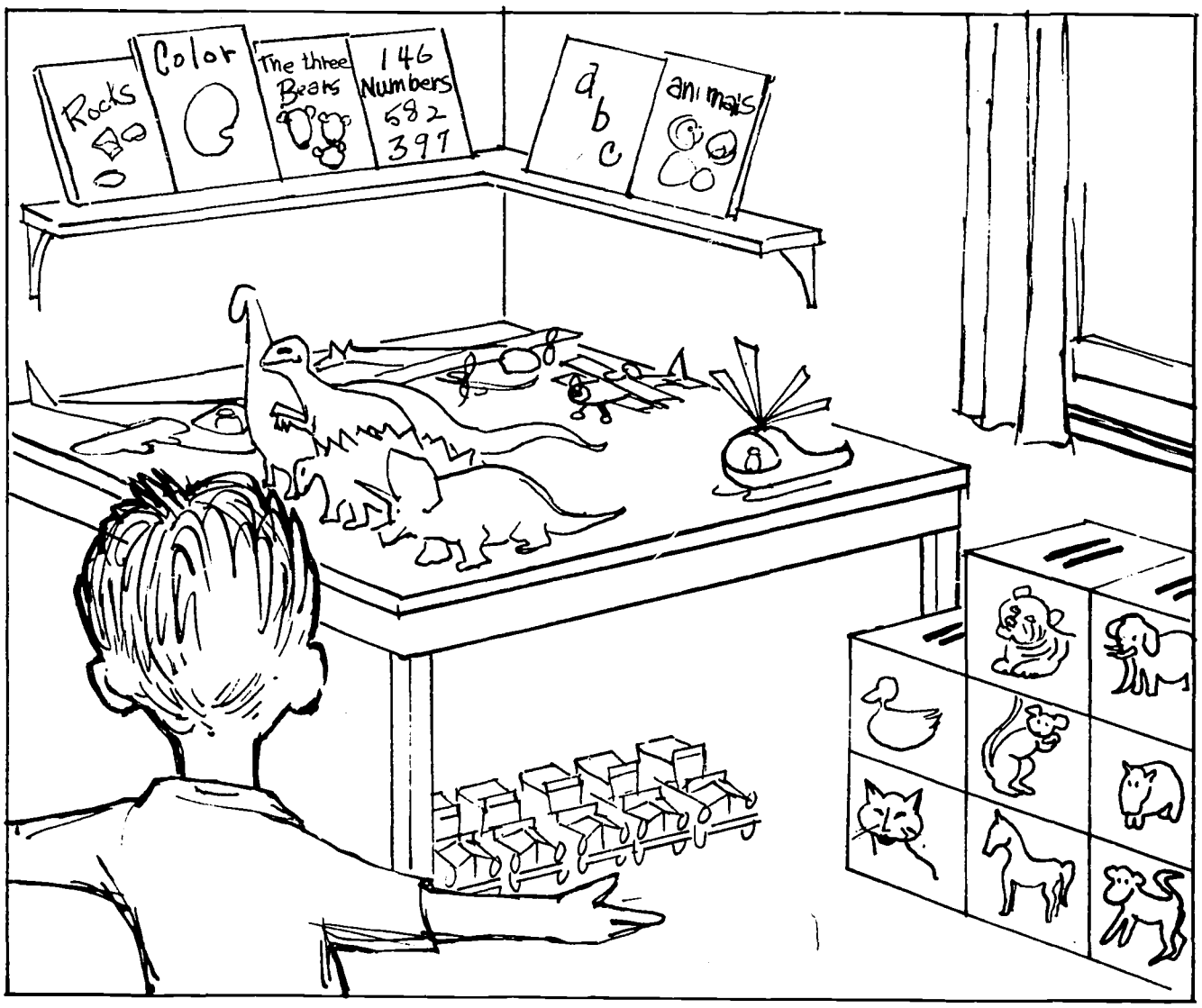


"But where shall I put my toy taxis and their drivers?"

David puts his hands on his hips and just looks at the new way he has arranged his toys. "This is the best clean-up job I've ever done," he laughs. "What a great surprise for Mother."

Here's a picture of David's room after he had put all his toys in sets. Can you find the set of dinosaurs? Can you find the Stegosaurus with the spikes on his tail? (This is the one Daddy stepped on.)

Look at the set of blocks. Find the set of planes and pilots in David's hangar. Do you see the set of taxis in their place?



Here's a picture of David's room after he had arranged all his sets of toys.

David is still looking around the room at his neatly arranged toys when he hears his friend Ricky's voice call, "David."

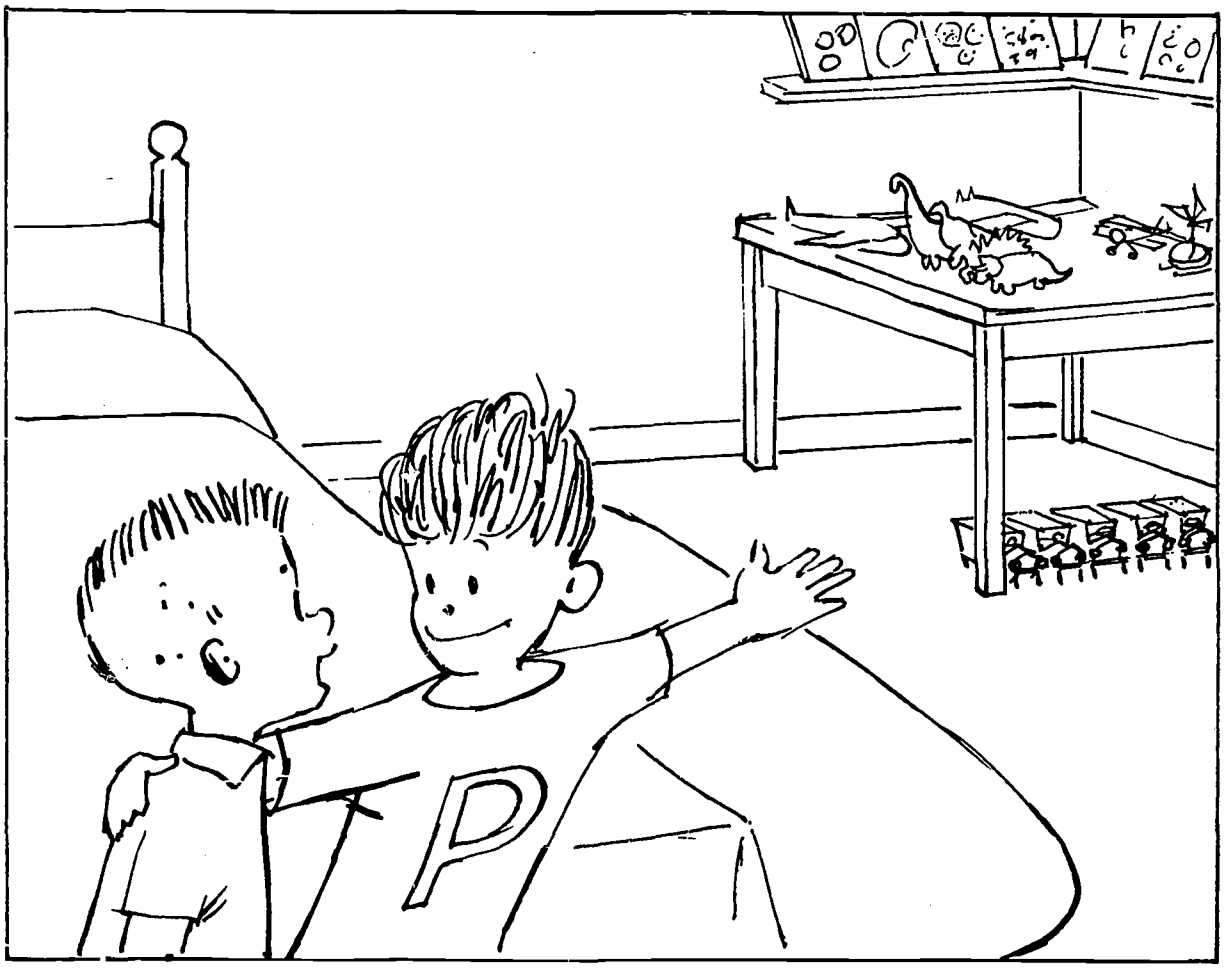
"Come in, Ricky," David calls. "Hurry and see what I've done!"

As David's friend Ricky (whose real name is Richard) comes skipping in, David exclaims, "Look! It's a surprise for my mother."

"Wowie!" cries Ricky as he looks at the sets of toys. "Will she ever be happy!"

Ricky glances at the neatly arranged toys, then at David, then at the toys, and again at David. Finally he asks, "Can't we just play a <u>little</u> with the toys before your mom comes home? We can make sure we put them back when we are through."

Can you guess what happens then?

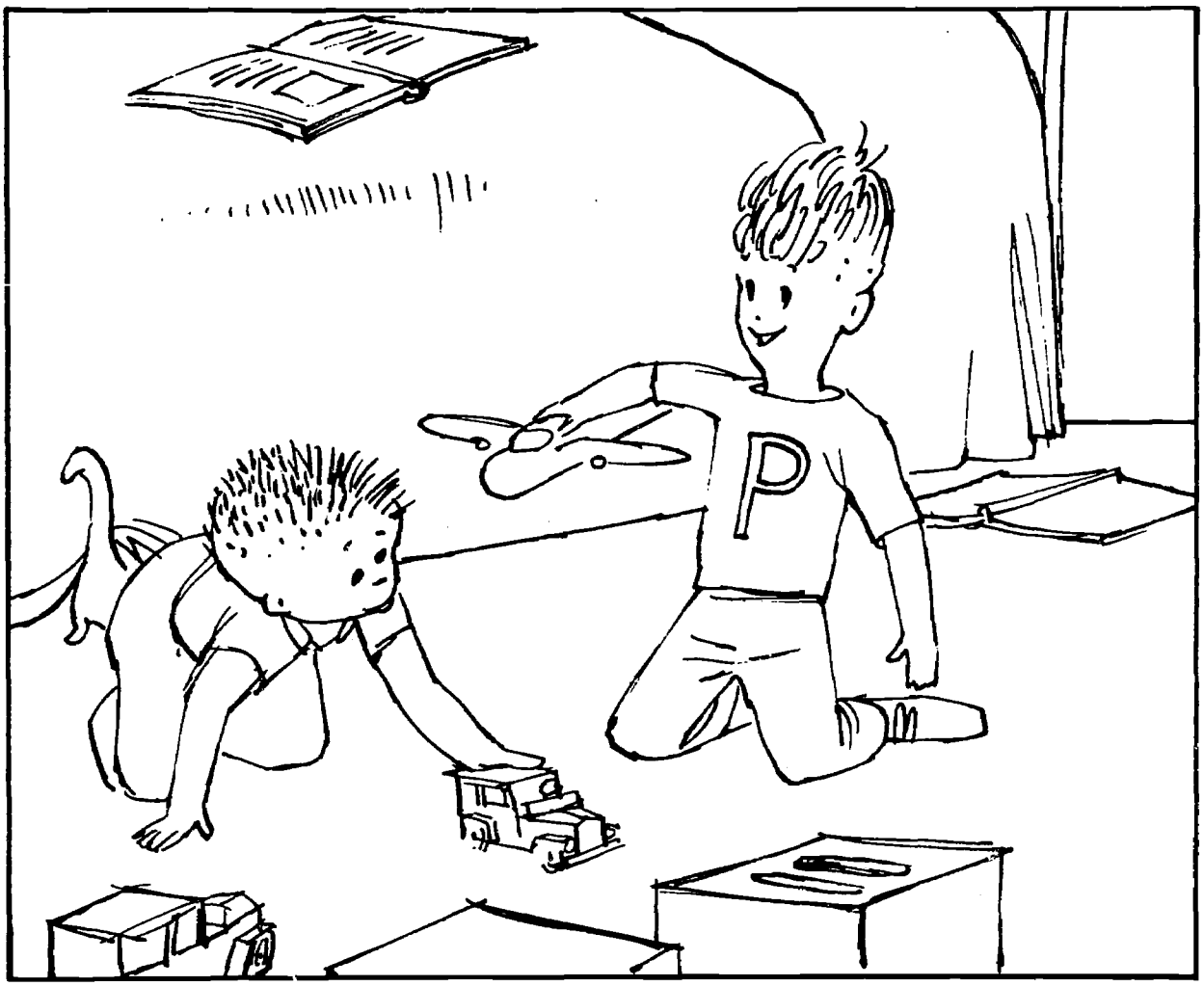


"Wowie!" cries Ricky as he looks at the sets of toys. "Will she ever be happy!"

Soon the taxis are speeding across the floor and block houses are toppling down on the dinosaurs, and a pilot is even sitting inside David's ABC book.

David and Ricky are having so much fun that they forget all about Mother's surprise. Suddenly David remembers and cries, "Ricky! We've forgotten about the surprise! And just look at the mess we've made!"

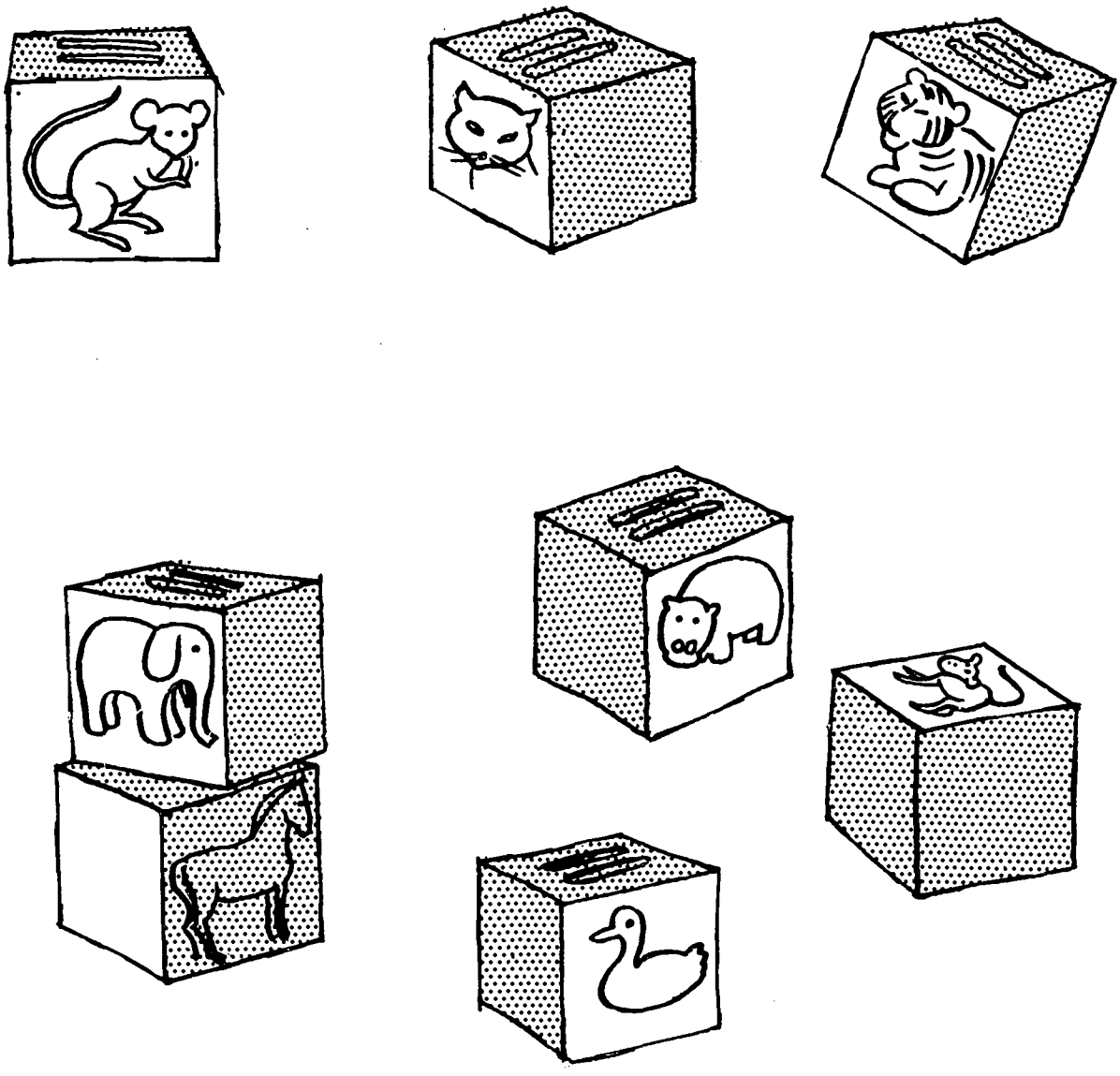
Quickly two sets of hands and two sets of feet began picking up toys. David starts with the books. They are everywhere.



David and Ricky are having so much fun that they forget about Mother's surprise.

"It looks like it has been raining blocks!" exclaims Ricky as he begins gathering them. "It seems like there are more blocks now than there were before."

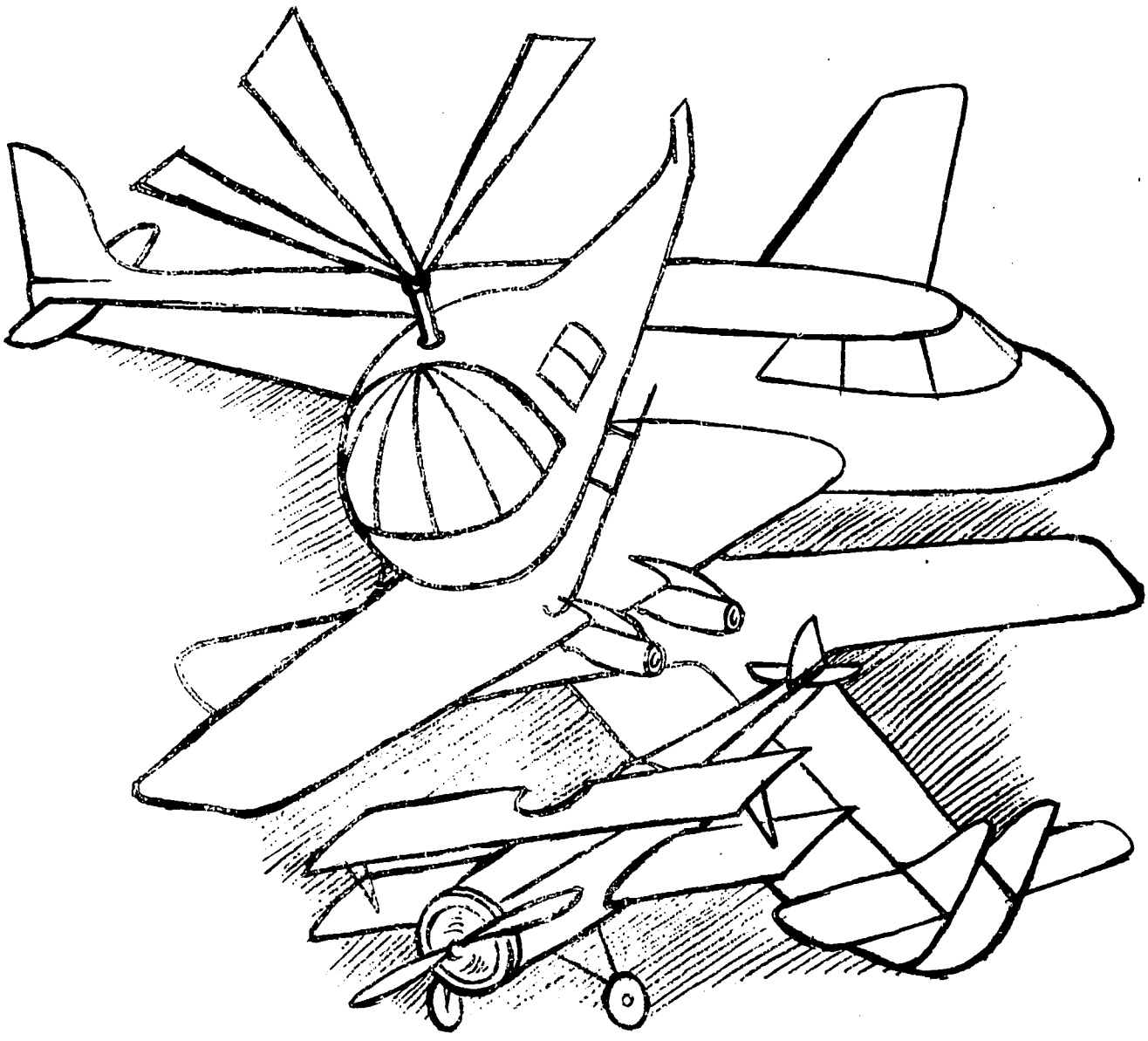
Do you think so? Let's see. Where is the block with the duck on it? The block with the elephant? The block with the horse? The one with the tiger, the cat, the mouse, the hippopotamus, the monkey? Does he have them all? Does he have the same set of blocks he had before? How can you be sure? It really makes no difference how or where we place the blocks, they are still the same set of blocks.



It really makes no difference how or where we place the blocks, they are still the same set of blocks.

Ricky looks worried. "David, what's happened to your airplanes? I see a little pile of them here but we must have lost some. Let's see. Here is your bi-plane, helicopter, cargo plane, and jet plane. I guess we do have the same set of planes we had before. When they were piled so close together it looked like there were some missing."

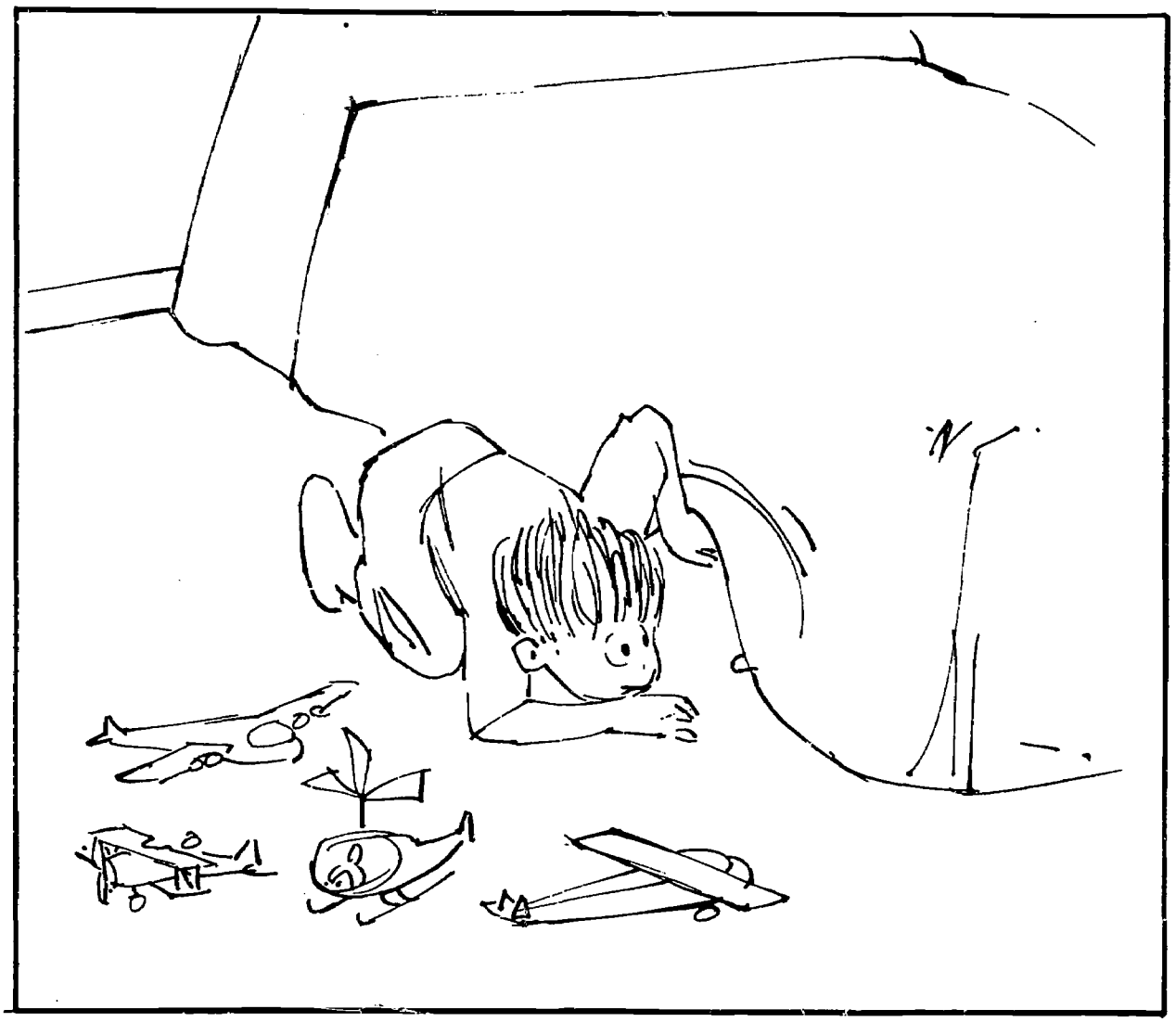
Does Ricky have the same set of planes he had before? How can we make sure?



Does Ricky have the same set of planes he had before?

"Ricky, we have all the planes, but where are all the pilots? We had a pilot for each plane and a plane for each pilot. Now, there is a pilot in every plane except the cargo plane. We have to find him."

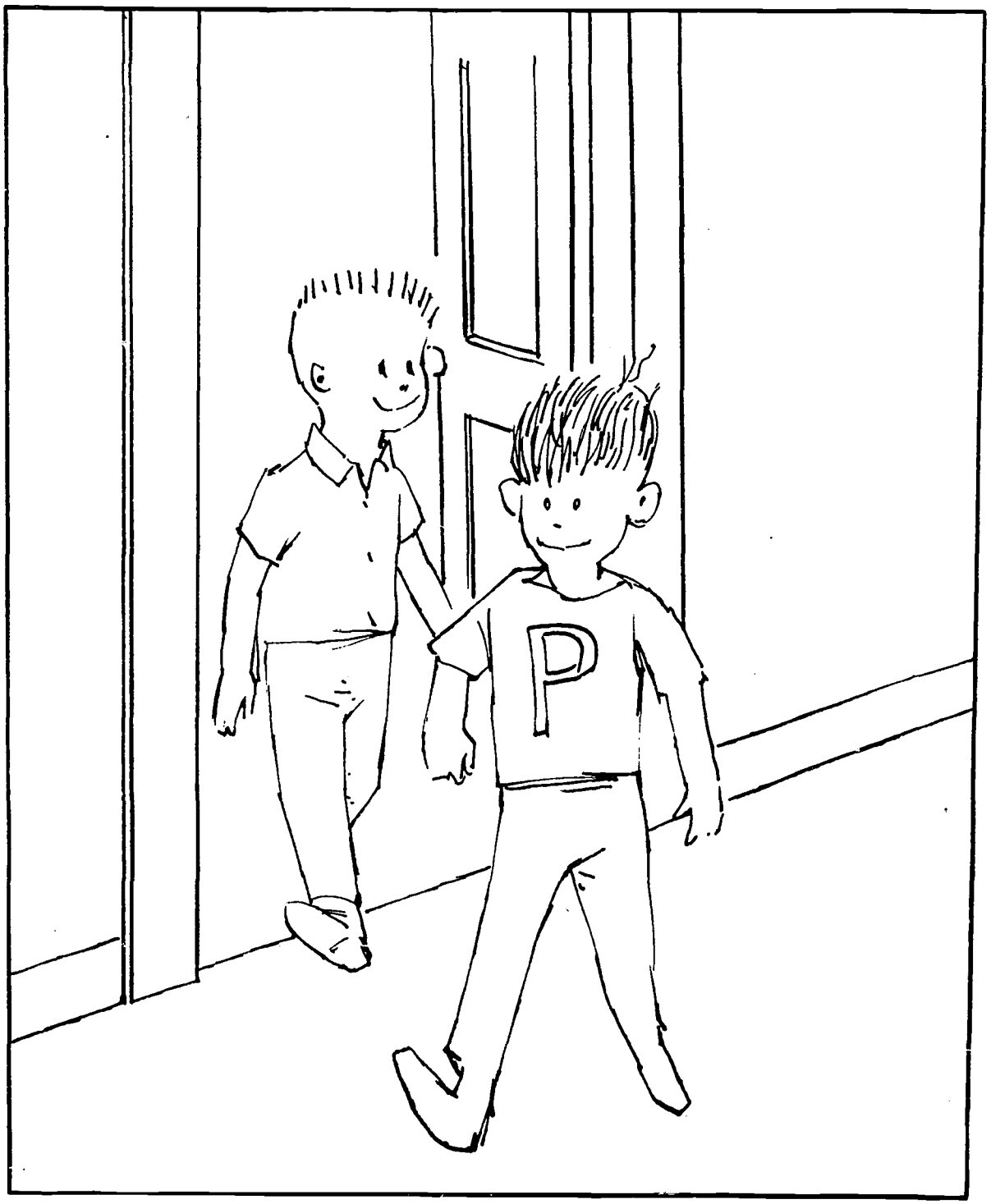
After some hurried shuffling and scrambling David shouts happily. "I see something under the bed and I think I know what it is. Look, it's the pilot for the cargo plane. Now every plane has a pilot!"



"I see something under the bed and I think I know what it is."

Soon David and Ricky have the taxis and drivers back where they belong, too. Just as David and Ricky sit down to rest, they hear the clickety-click of mother's high heels on the sidewalk. "It's Mother," David exclaims joyfully. "We just made it!"

"Let's go to meet her, Ricky."

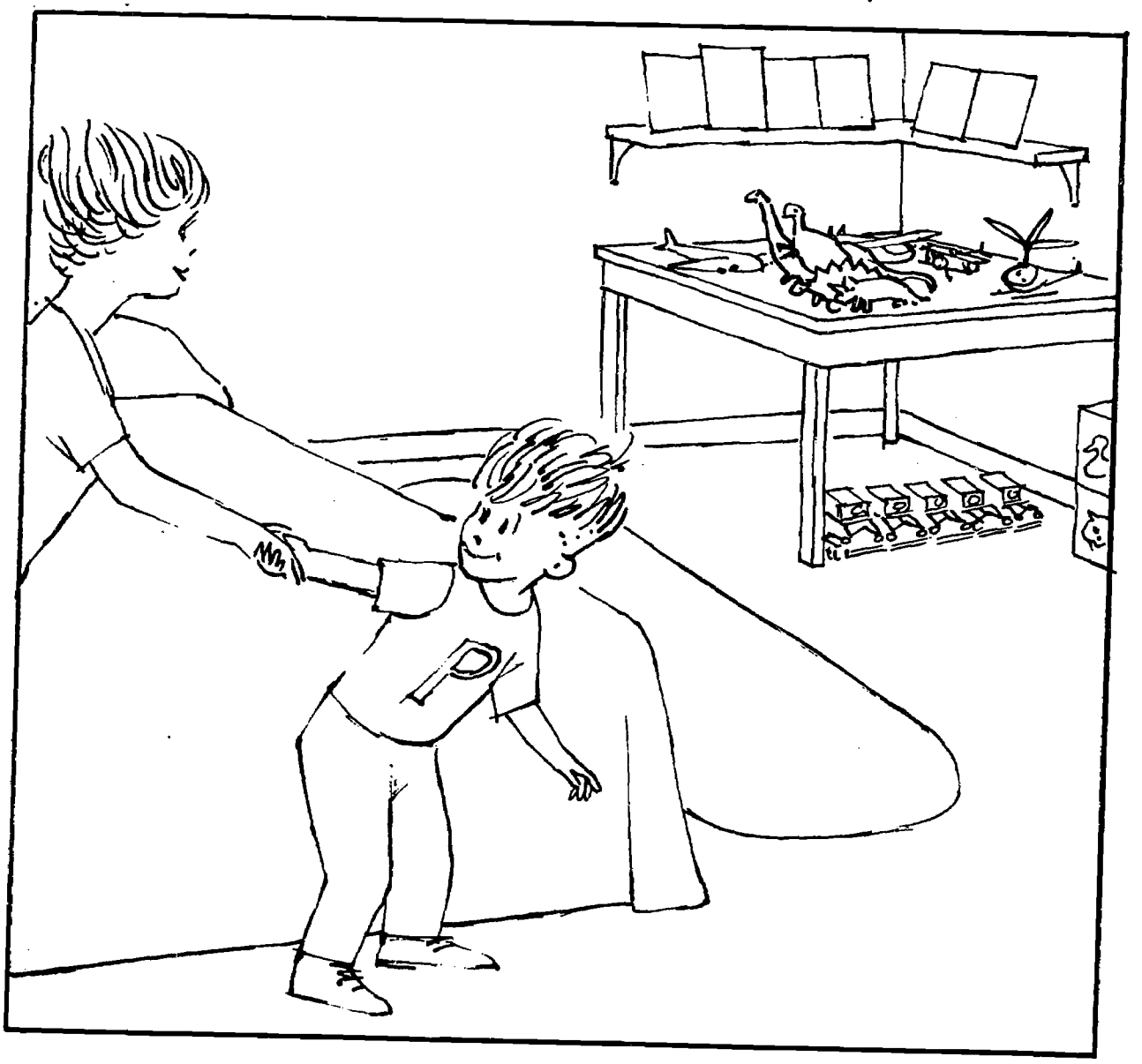


"Let's go to meet her, Ricky."

David grabs his mother's hand and pulls her towards his room.

All the way to the door he talks excitedly, "We have a surprise, Mother, come and see it! Now daddy won't have to worry about spiky dinosaurs on the floor, and Kathy will have plenty of room for her schoolbooks, and see, Mother, I really <u>am</u> keeping my promise!"

DAVID'S PROMISE

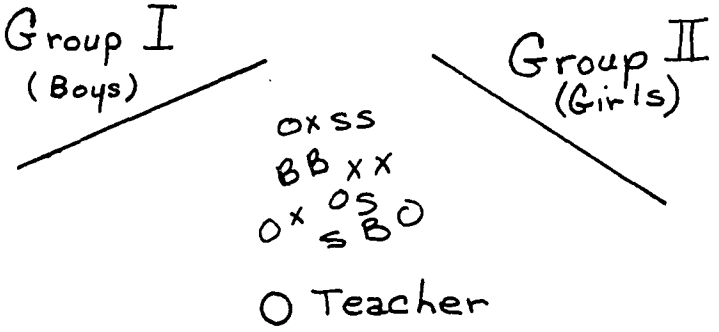


"Look, Mom, I kept my promise!"

Commentary on the Song and Activity "Get Set"

1. Divide the class into two groups. The words suggest a boy/girl division.

2. Place the set of toys, books or other appropriate items in a random pile between the two groups of children. Feel free to substitute your own sets of items, both in the song and activity.



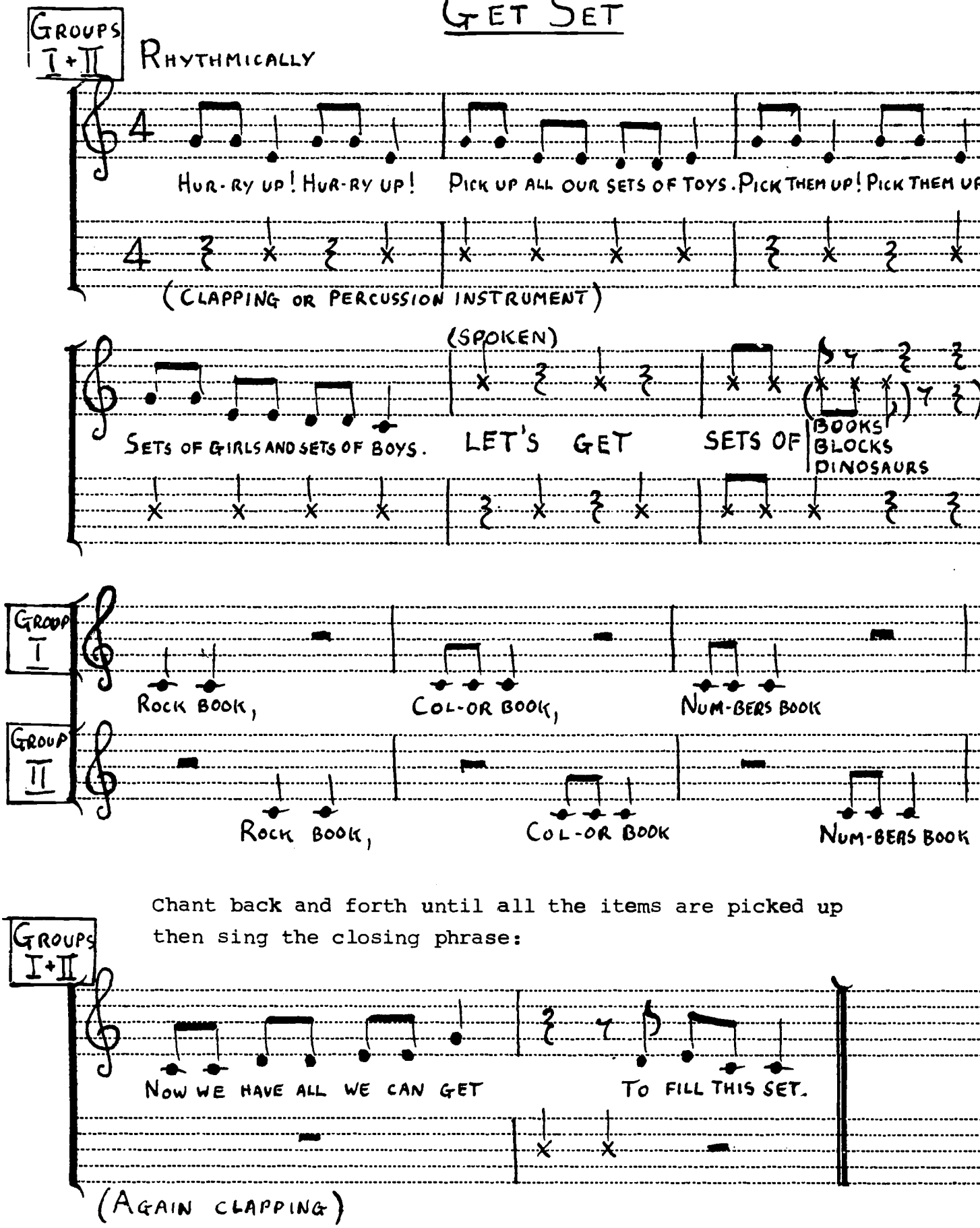
3. A child from each group is assigned to select, in turn, items from the pile of objects. As each item is selected and held up so that it may be seen, its name is chanted by one group, then echoed by the other. Excitement is generated by picking out the items fast enough to maintain a lively, even tempo.

4. Initially, the teacher should select the first few items for demonstration purposes, until the children are familiar with the game procedure and the song.

Commentary on Worksheet 1 - Members of a Set

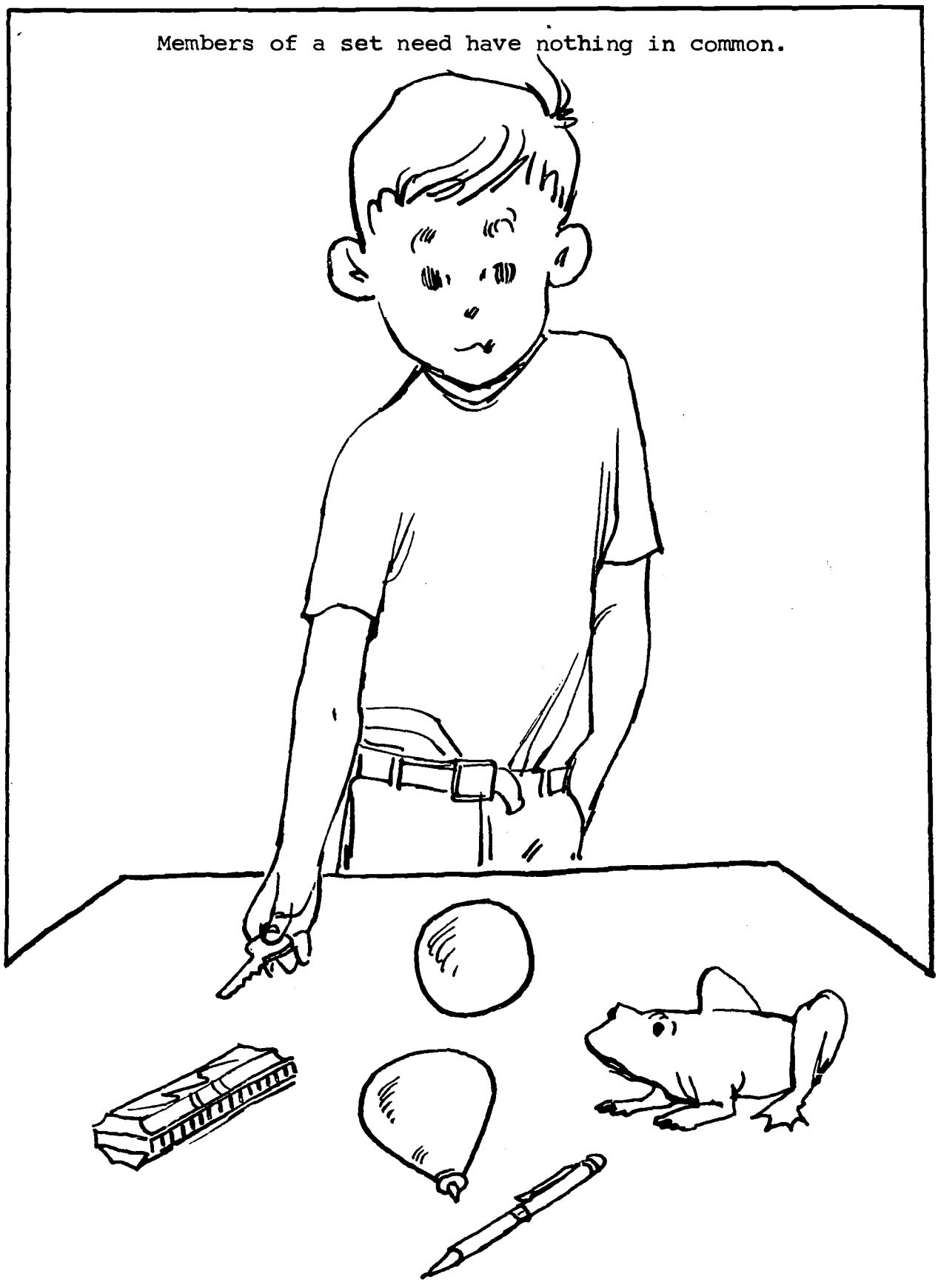
Worksheet 1 illustrates that the members of a set need have nothing in common. Discuss the worksheet with this concept as the focal point.

GET SET



Chant back and forth until all the items are picked up then sing the closing phrase:

Members of a set need have nothing in common.



Suggested Activities on Conservation of Sets

Note: Caution is necessary when using concrete objects to illustrate set and its properties. For example, except under an abstract interpretation, a set containing a pencil, a paper clip and a rubber band is not equal to (or the same as) a similar set of a pencil, a paper clip and a rubber band, even though the two sets are so alike that you can't tell them apart. Here the sets are equivalent, but not equal. The fact that two paper clips are not the same concrete object would be obvious if a child swallowed one of them.

Also, it is not possible to simultaneously display two equal sets without using some symbolization or abstraction. The difficulty of this is obvious when you consider such unique objects as people!

1. To show conservation of sets, display an empty shoe box with a cover and let the children check it to be sure that there are no holes in it.

   Next ask various members of the class to suggest things such as blocks, pencils, crayons, buttons, etc. that you could put into the box. It is best to keep the number of items small, about 4. All items should be easily distinguishable from each other.

   As each object is placed into the box, have a child keep a record of it by drawing a picture of it on the chalkboard.

   Next, place the cover on the box and shake it until all of the objects are well mixed. Now open the box again,

tip it on its side so the class may see what is in it, and ask them if the set of objects in the box now is the same set of objects that was in the box before you shook it.

* 2. "Pass the Object"

Children are seated in a circle with the teacher. Each child is given a block, toy, ball, or other convenient object. There should be no two objects alike. At a signal each child passes his object to the child at his right. Children might chant:

"Pass it to the right,
Pass it to the right,
When the signal comes
We'll hold on tight".

At the word tight, everyone holds on to the object or objects in his hands at the time. (Or teacher might play piano or phonograph as a cue for the children to hold on to the objects they have when the music stops.) Ask:

"Do we have the same objects we had at the beginning of the game?"
"Do we have the same set of objects we had before?"

The teacher should remove or add objects as the teacher and children pass them from one to another. This will bring out a greater variety of responses.

3. Demonstrate the concept of conservation of sets by stacking blocks in various shaped piles, putting them into a basket or placing them on a line, etc.

4. Give each child one object. Ask:

"Does each child have one object?" (YES)

"Does anyone have more than one object?" (NO)

Have the children walk around in a circle and drop their objects into a box or basket in the center of a circle, and then sit down again.

"Do we now have the same set of objects as we had before we put them in the box?" (YES)

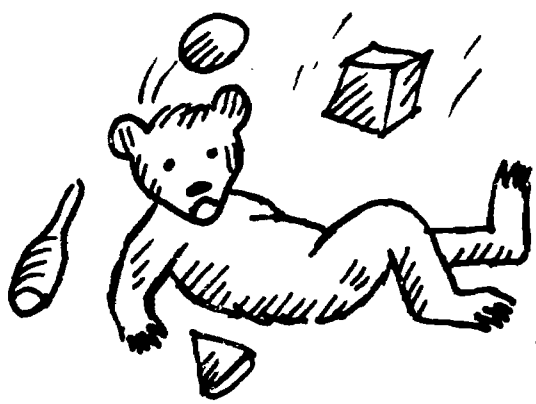
"Do you suppose that there is still one object in the box for each person?" (YES)

"How can we tell for sure?" (We could give each person one block from the box.)

Commentary on Worksheets 2, *3, - Conservation of Sets

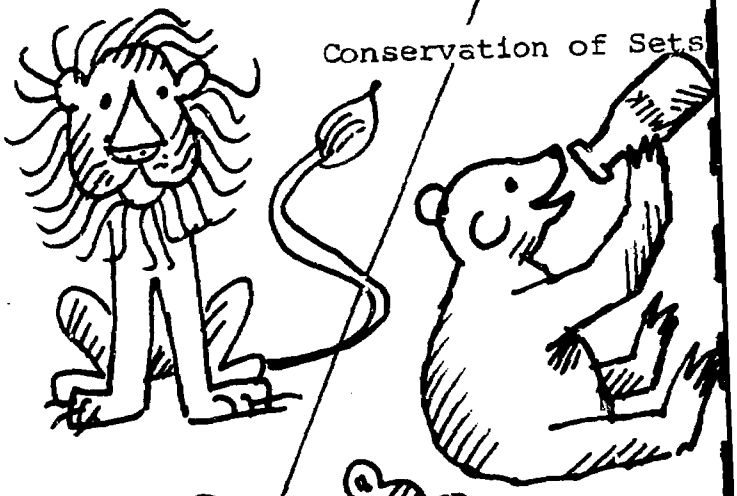
These two worksheets deal with the conservation of sets under rearrangement of the members of the sets. The worksheets emphasize the fact that the order in which the members of a particular set are arranged does not affect the membership of that set. The children are to draw a line from the set on the left to a picture of the same set on the right side of the worksheet.

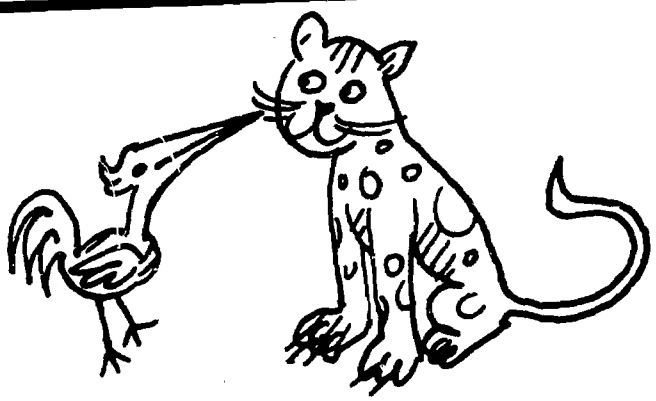
Worksheet 2

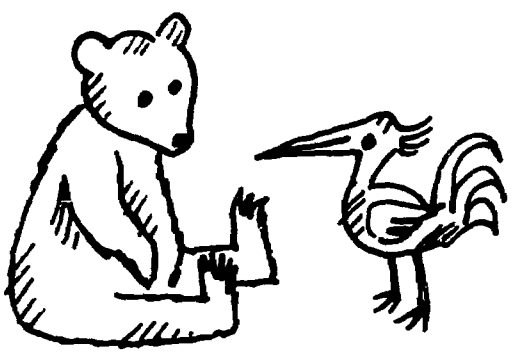


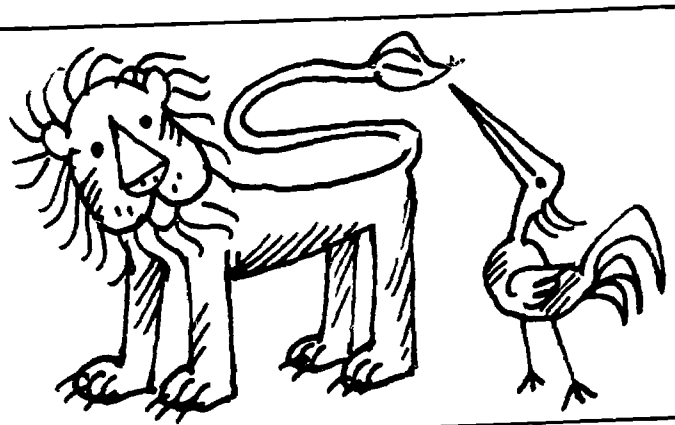
Worksheet 3

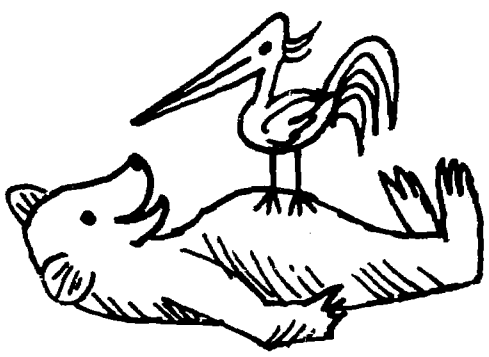
Conservation of Sets











Teacher Background on Empty Set

Most people considering set for the first time would probably assume that this term would not be used unless at least two members were involved. Furthermore, they would probably not consider a set containing a single member as a set and instead, would consider only the characteristics of the member itself. However, mathematics does not make these restrictions. Set, as used in its mathematical sense, extends to one or even no members. The following will illustrate.

Consider a local Garden Club as a set. For a while, the local organization flourishes. Gradually interest diminishes and soon there is only one member who has not resigned or moved away. At this point, the local club is a set of one (Mrs. Brown). Finally, Mrs. Brown moves away and the club becomes an example of the empty set, a set with no members.

It is possible to describe the empty set without making any reference to the concept of number. Examples such as "the set of children with green hair", "the set of all elephants in this classroom", or "the set of books that have no pages" are interesting and appropriate. Reports from teachers experienced in introducing the empty set indicate that children are ingenious and original in thinking up their own descriptions of the empty set.

It may seem strange to say that the "set of boys named Mary" is the same as the "set of all elephants in the classroom", but these sets are the same, since neither contains a member which is not in the other (neither contains a member!). Although there are many descriptions of the empty set, there is only one empty set.

There are several reasons why it is important that students acquire an early understanding of the concept of the empty set. It has an immediate application in the development of the concept of the number zero. The empty set also plays an important role in the set interpretation of addition.

Suggested Activities on Describing Sets

Note: In considering sets, examples such as "the set whose members are a table, chair, book" are commonly given. The person giving such an example usually does not care precisely what set his audience thinks of. To a person just learning about sets, such examples are more misleading than helpful, since they cloud the fact that sets are different if they contain different members. If two people think of two different chairs, then they are not thinking of the same set. To define the set adequately it might be necessary to say "the set whose members are the red table, the brown chair beside the window, and the blue book on the teacher's desk".

If a set is described properly, the person hearing the description should be able to point to the precise members of that set. If he cannot do this, then the description needs to be more specific.

For further reference see Teacher Background on Describing Sets, page 4.

The following activities also provide an opportunity for the children to use what they have learned from the Minnemast Science lessons on objects.

1. Suggest to the children that it is easy to describe many sets in the classrooms, such as a set of desks and a set of chairs. Ask them to describe other sets in the room (books, toys, games, records). Have someone locate the members of these sets.

2. Say "We have a set of children in this classroom. Raise your hand if you are a member of this set." (All children should raise their hands because they are all members of this set.) Each of you is a member of this class, so you are a member of this set."

3. Ask this question: "Can you think of a set in this room that has only one member?" Many answers are possible -- flag, teacher, pencil sharpener, teacher's desk, round table, phonograph, movie screen, wall clock, etc.

4. Have children name sets they have at home. Examples are family, toys, dishes, etc.

5. We can define a set by stating its membership requirements. For example, "the set of girls in the class who are wearing something blue". The set of girls wearing something blue is the set whose members are Maria, Peggy. Note that the order of naming makes no difference; Maria, Peggy or Peggy, Maria.

6. Have the children suggest as many words as they can that are used instead of set or that describe some special kind of set. Have them ask their parents for suggestions and bring a list of these to school. Compliment the child who has the most words on his list. Commend, also, the child who brings or thinks of a word that no one else has.

The following are appropriate examples:

group	family	pair
collection	flock	trio
bunch	herd	quartet
pile	team	score
heap	class	brace

7. Each child should be encouraged to bring a set. Who can bring in the most unusual set?

* 8. Play the game "Find the Set".
The purpose of the game is to dramatize the importance of accurate description of a set. It is not a guessing game, but an exercise in describing the set in such a way that any child may immediately point them out. The challenge in the game is to the child describing the set, rather than to the "finder".

Have the child who is chosen to be the finder cover his eyes or leave the room while the class decides on the specific set for him to locate.

When the set has been decided, the finder returns and chooses a child to describe the set to him as clearly as possible. (The child describing the set should be cautioned not to give away the position of the members of the set by looking at them.)

If the "finder" recognizes the set from the description given, he goes over and points to each member in it. If his guess is incorrect, he may choose another child to describe the set for him.

The child who successfully describes the set becomes the next "finder".

9. Use the following examples of sets with no members:

Ask all girls who belong to a Boy Scout troop to clap their hands.

Ask all students who did not go to bed the night before to wave their hands.

Ask all boys in the class who have green hair to stand up.

Select a student to go around the room and collect all the hundred-dollar bills.

* 10. Call on the children to give a description of the * <u>empty set</u> which has not yet been given.

11. Ask that all of the students who are in both kindergarten and first grade to stand up. The fact that no child can be in both grades at once may be hard for the children to grasp, and usually follows only after considerable experience with the empty set.

12. Conclude this topic by restating the principle that although all of the sets above are described in different ways, the descriptions all refer to the same empty set.

Teacher Background on One-to-One Correspondence, Subset

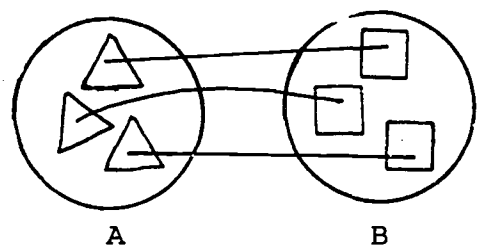
One important way of comparing sets is to match their members by one-to-one correspondence.

Most children have had natural exposure to the idea of one-to-one correspondence between the members of two sets. As a pre-schooler a child is given a cookie for each hand; there is a plate set at the table for each member of the family; other experiences have utilized the matching process.

In comparing sets in this way, we pair up one member of one set with one member of the other set. If such pairing consumes or uses up all of the members of both sets, then there is said to be a one-to-one correspondence between the sets.

Reminder: Sets are equal only if they have the same members. While the establishment of a match between two sets shows them to be equivalent, it does not make them equal. If we state that two sets are equal, we are simply using two different names for the same set.

Example

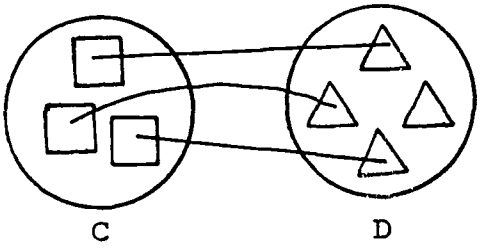


Because A and B can be matched in one-to-one correspondence, they are equivalent sets and we would write:

A is equivalent to B, or A matches B, or B is equivalent to A.

Symbolically, $A \sim B$ or $A \leftrightarrow B$

Two sets either can be matched in one-to-one correspondence or they cannot be matched. If they are non-matching sets, then one of the sets has more members than the other one.



Sets C and D above are non-matching sets. We say that D has more members than C or C has less members than D.

The terms more than and less than will be used throughout the kindergarten units when referring to the results of matching sets. The mathematical term less than is preferred to fewer than. The teacher ought to be aware of this preference, and use it consistently, but need not over-emphasize its use by the children. If a child occasionally uses the more common term, fewer than, but in a correct context, the teacher may consider it acceptable.

The teacher should avoid using the terms larger set or smaller set. To the children these terms may imply size of members, rather than the number of members.

Note: The terms "matches" or "matching" are used with the children, but we frequently use "equivalent" in communicating with the teacher. The use of the expression "equal in number" is delayed until Unit IV, since we prefer not to mention the term number at this time.

Subset

The term subset is used to indicate a relationship between two sets. It means that one of the sets is a part of the other set. For example, if E = {Tom, Bob, Bill, Ray} and F = {Tom, Bill}, then F is a subset of E.

Since the expression "subset of", like the terms "more than" and "less than", indicates a comparative relationship, it is pointless to say something is a subset unless the context indicates what it is a subset of. Merely saying "F is a subset" is like saying "1,000,000 is less".

Every nonempty set has at least two subsets - the original set itself and the empty set (the empty set has one subset).

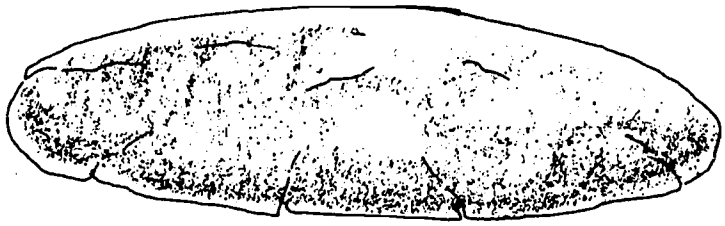
This is best exemplified by citing one of the activities which appears later in this unit. We say to the children, "Here is a plateful of cookies. Put an X on each member of the subset you'd like to eat." Many of the children will probably indicate all of the cookies, and this is appropriately termed "the subset of cookies you'd like to eat", even though it's the entire set.

We also ask them to indicate the set of cookies which they would not like to eat. In most cases this turns out to be the empty set; it too is a subset.

Many adults have difficulty in accepting the mathematical fact that a set may be a subset of itself. This is probably because of the usual association with the prefix "sub". Fortunately, children are not limited by earlier, contradictory learning and are able to readily accept this concept. The fact that the empty set is a subset of every set also proves troublesome to some adults, though not to children.

The story "Matching Cookies" dramatizes the use of one-to-one correspondence and subset.

MATCHING COOKIES



Comparing Sets by
1-1 Correspondence;
Subset

School had only been out for a week, but already Bill was bored. The first week it was wonderful to do nothing but play with his brother, Pete, who wasn't old enough to go to school. But now! Bill was just plain tired of playing with his small brother. He had read all his old comic books. He had swung until he got so dizzy he couldn't walk straight. One day he even practiced his music without his mother making him. That meant he was <u>really</u> bored.

One morning Bill was lying on the floor in front of the T.V. set. He was watching a cartoon show that he had already seen four times. Bill didn't even bother to look up at Mother when she walked into the room.

He had swung until he got so dizzy he couldn't walk straight.

"Bill, wake up!" called Mother. "Listen while I read you this letter from your Grandmother." Mother began to read, "Dear Daughter, I wonder if I could persuade you to lend Pete and Bill to me for two weeks. I think they might enjoy the country for a while. Their Grandfather will have a gently pony that our neighbor is loaning us while he is on vacation. The pony will be here next week and the week after that.

I understand that Bill is working on his cooking badge for Cub Scouts. Perhaps I can help him earn that badge.

Let us know if the children are coming and how they will travel if they come. Just tell us when to meet them."

Bill wasn't bored any longer. He jumped up. He waved his arms. He even tried to stand on his head. Pete heard the noise and came running. Everything Bill did Pete tried to do, too!

Bill wasn't bored any longer.

"We're going to Grandma's! We're going to Grandma's!" shouted Bill.

Next Monday the stewardess on the Hi-Fly Air Lines led the two excited children from the plane. They raced into their waiting grandparents' arms.

What wonderful, exciting days followed! Pony rides, swims in the creek, trips to the country store with their grandfather.

Then Grandma said one morning, "Today Pete and Bill and I will make cookies." So into the kitchen they went. Bill helped Grandmother measure flour, sugar, spices, shortening, and molasses. Pete cracked the eggs that were needed, and only one slipped away from him and made a mess on the floor. Grandmother and Bill agreed that that was very good indeed for such a small boy.

What wonderful, exciting days followed!

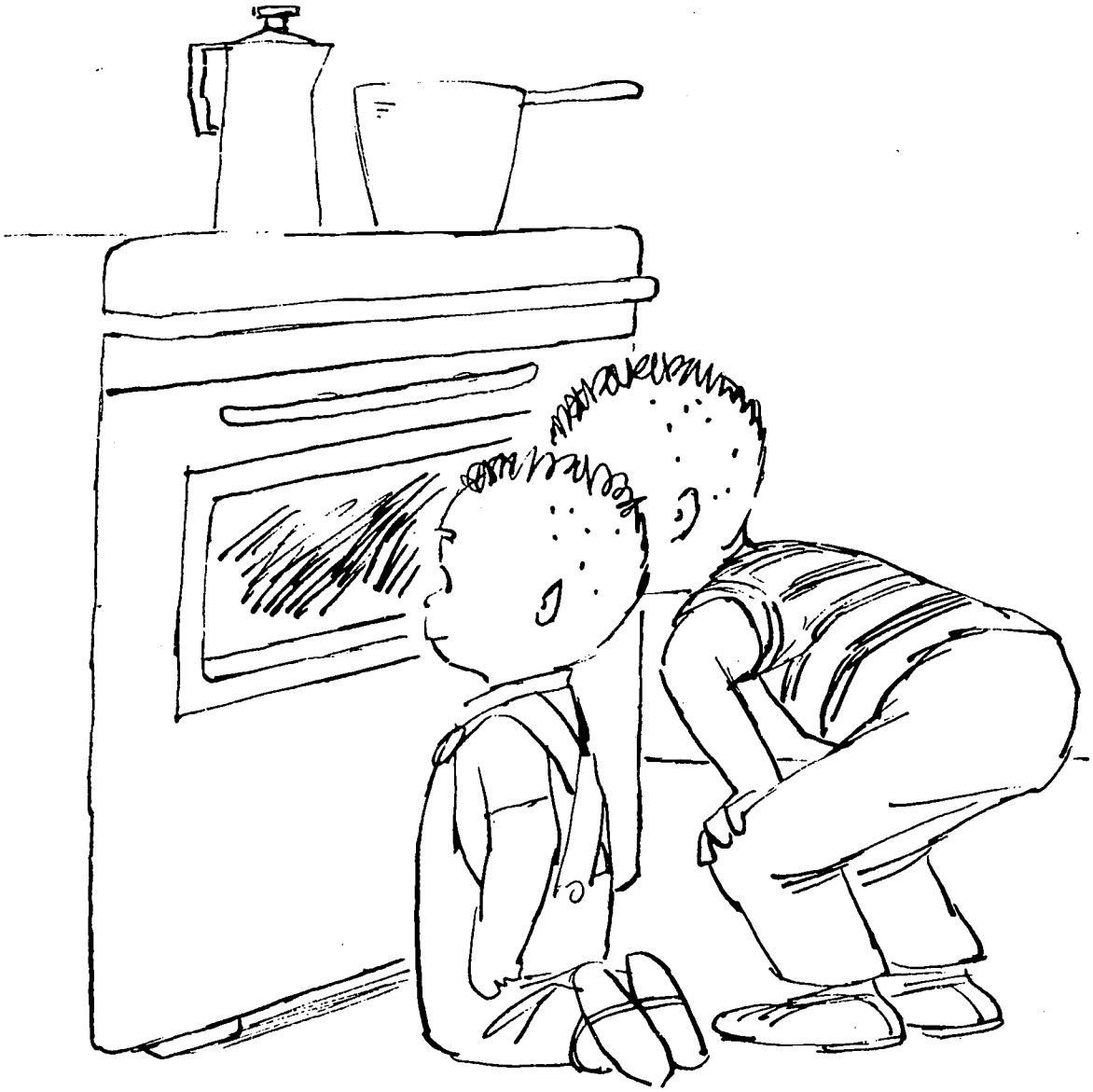
At last the spicy dough was finished. Grandmother gave the boys some dough to mold into cookies. Each boy had a baking sheet to fill.

Soon the sheets were covered with cookies and ready to be baked. Grandmother put Bill's pan on the top shelf in the oven and Pete's on the second. The two boys watched through the glass front of the oven as the cookies baked. The cookies looked almost as good as they smelled and that was very good indeed.

And then, none too soon, the cookies were baked and cooling on the two racks. After the boys had each eaten two of the crispy cookies, they put the rest of the cookies into two bags. Bill put his cookies into one bag and Pete put his cookies into another bag. Then they went into the dining room. There the boys opened their bags and looked at those beautiful cookies. They sniffed the wonderful spicy smell.

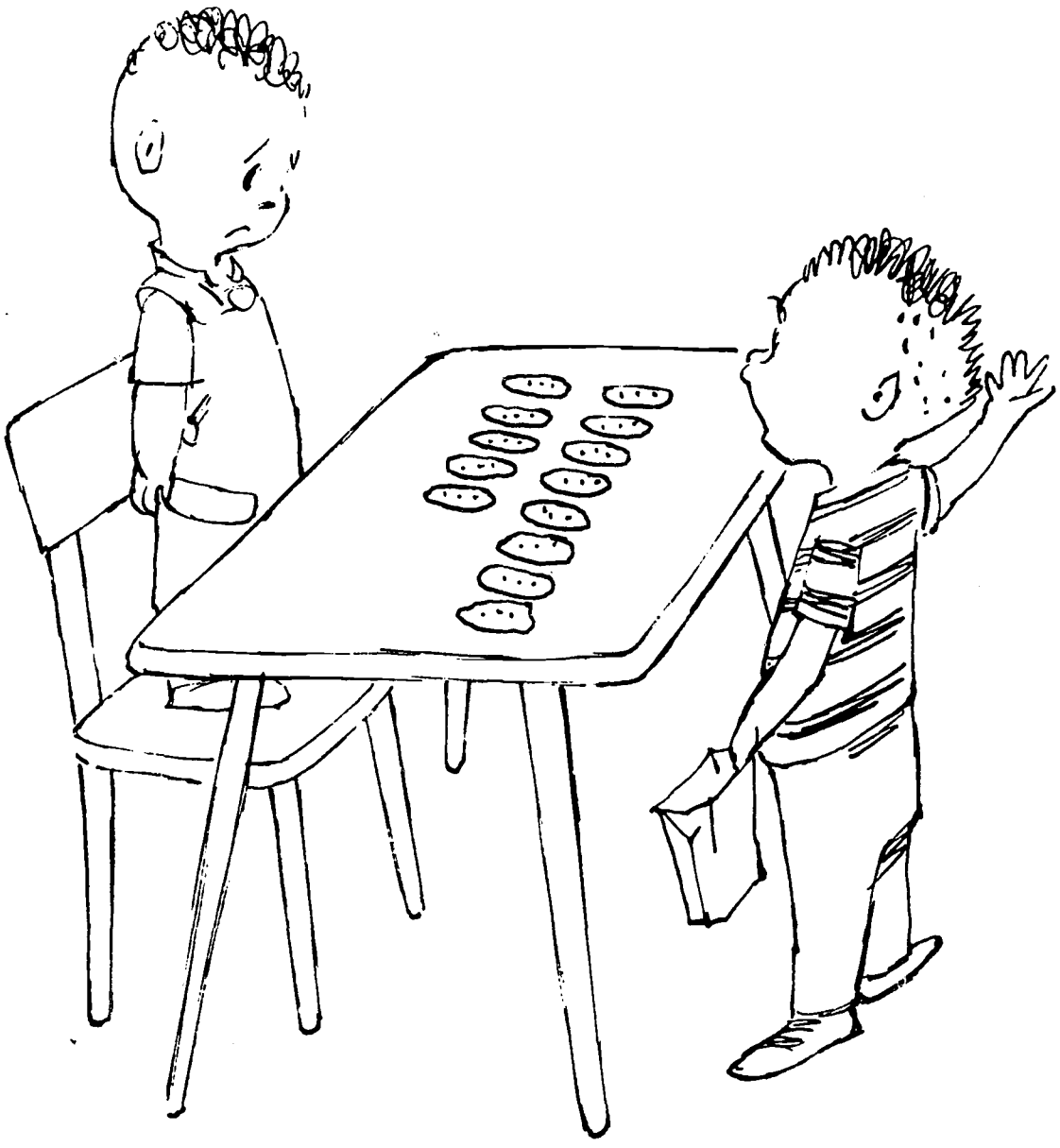
Pete, looking into his bag, chanted, "Cookies! Cookies! I have the mo-ost! I have the mo-ost!"

Bill looked into his bag and said, "No! I know I have the most."



The two boys watched through the glass front of the oven as the cookies baked.

And then Bill said, "Let's line our cookies up on the table. I'll put mine right here. Each time I put one down you match it with one of yours right beside it. O.K.? Here is the first one for me." Bill put his cookie down on Grandmother's nice clean tablecloth. Pete matched his first cookie with Bill's. Bill put down another cookie. Pete matched it with one of his cookies. And so they continued. Each time Bill put down a cookie from his sack, Pete matched it with a cookie from his. Finally Bill put down a cookie, and Pete reached into his empty bag. He let out a disappointed squeak, "I don't have any cookies left."



"Let's line our cookies up on the table."

Then Bill pulled out another cookie and still another while Pete gave disappointed groans. Bill said, "I have more than you because I have a cookie for every one of yours and I have a subset of cookies left over.

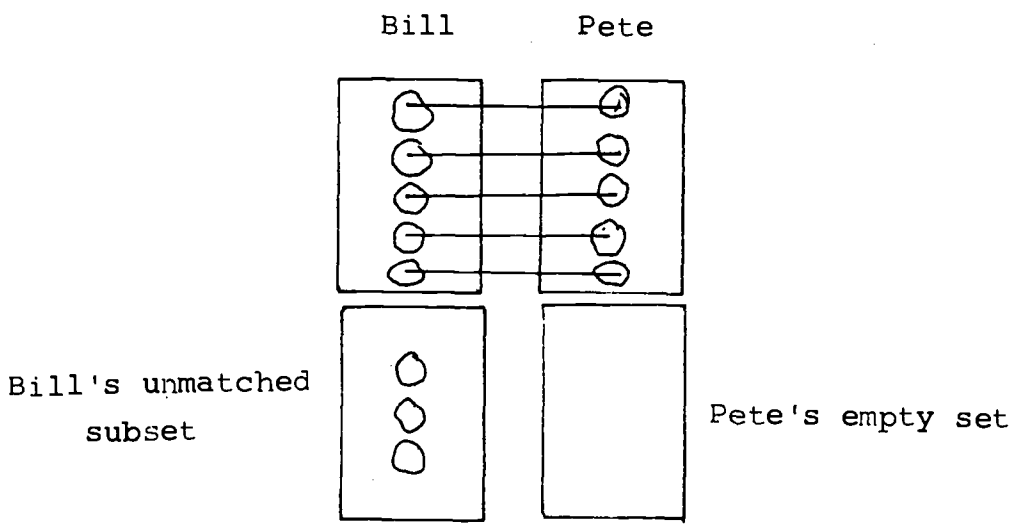
Poor little Pete looked down at the table. He said, "I don't have a subset left, do I?"

Bill felt sort of mean and then he remembered what he had learned in first grade last year. "Don't feel bad, Petey," he said. "You do have a subset left, too, but yours is the empty set. You see, Petey, since your subset is empty and mine has some cookies in it, you have fewer cookies than I. Now we both know who has the most. Let's see who can eat his cookies faster."

"Let me start! Let me start!" cried Pete. "I am really good at eating."

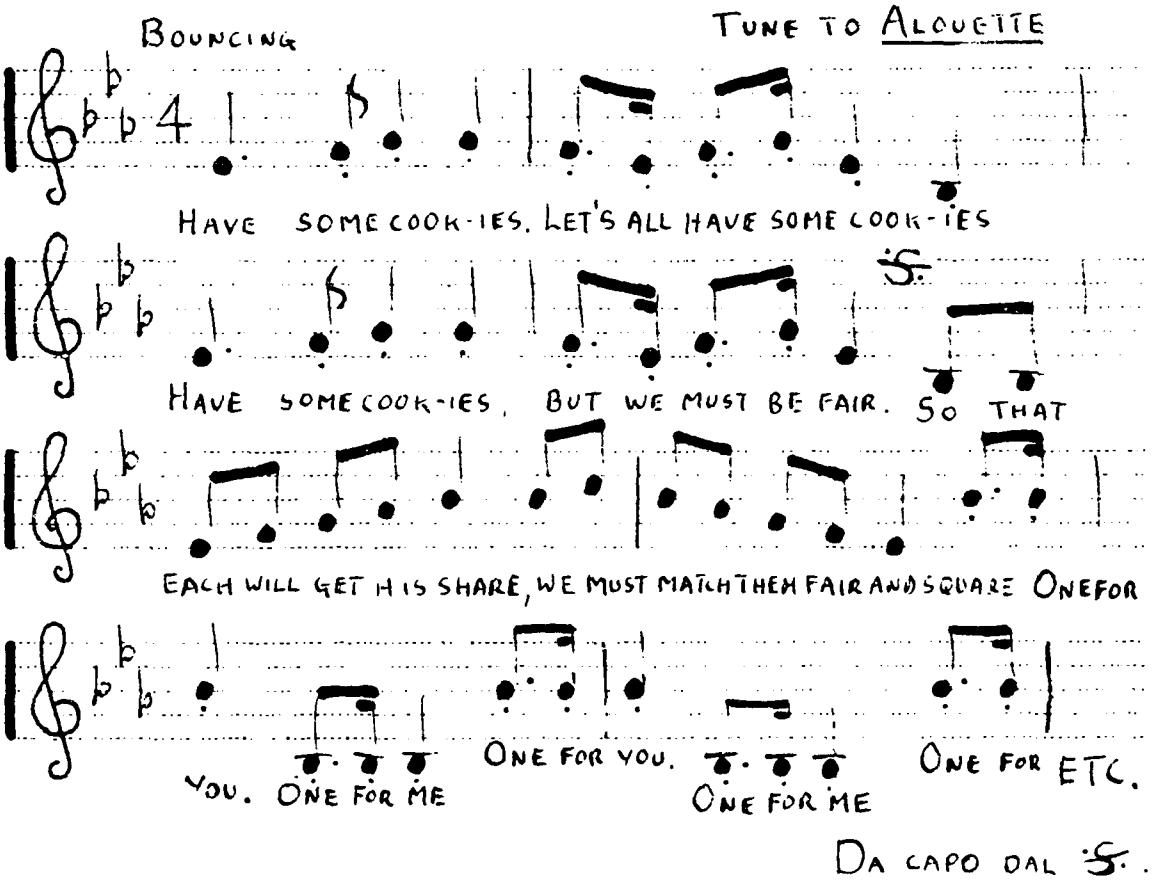
And Pete did finish his cookies first. Perhaps it was because he was a fast eater and perhaps it was because he had fewer cookies than Bill.

(Teacher may use this diagram in explaining the story.)



"Let's see who can eat his cookies faster."

THE COOKIE - LARK



Divide the class into groups, and sing as follows:

Group 1 "Have some cookies, let's all have some cookies,"

Group 2 "Have some cookies, but we must be fair."

Together "So that each will get his share, we must match them fair and square, etc."

Matching activities, using cookies, pebbles or other small objects are a natural accompaniment to the song. The children may be arranged in the original two groups or in individual pairs.

Teacher Background on Counting

Many children are able to recite the sequence of English number words up to ten and some up to one hundred. However, when asked to count the spokes on a wheel, to gather 17 blocks, or even to move eight spaces on a game board, they will frequently make errors.

The children do not understand the importance of one-to-one correspondence; their recitation of the words gets out of step with their "checking off" of the members of the set. Although it has great social significance, the memorization of the English number words has no mathematical significance.

The best way to overcome this is to present children with situations in which counting is difficult but direct comparison using one-to-one correspondence is easy. This unit has confined the content to that of comparing two given sets; the children are not asked to compare sets unless both are simultaneously available.

The question "How many?" and its answer have been delayed until Unit IV.

Suggested Activities on One-to-One Correspondence, Subset

1. Have two children pretend that they are Peter and Bill, as in the story "Matching Cookies". Give "Bill" a handful of stones, and "Peter" a handful of shells, to pretend they are cookies. (Beads, blocks, other objects will do.) Ask the class how they can determine which child has *more objects than the other, or whether the sets *match? The expected response is a suggestion to imitate the action depicted in the story, with Bill and Peter matching their objects by placing them side by side.

2. Ask if there are more children than children's noses in the room. Have the answer justified.

* 3. Ask if there are more pupils than chairs or more chairs than pupils. Discuss a way to find out. See if there are any pupils or chairs left. Ask the children if the result would be the same if they sat in different chairs.

4. Determine whether there are more boys than girls or more girls than boys. Have each boy choose a girl partner and see if there are any children left over. Ask if the result would be the same if different partners were chosen.

5. Show some checkers. Ask if there are more red than black, or more black than red. Find out by matching.

6. Divide a set of marbles between two children. Ask if Jim has as many as Ann. Try experiments in sharing among 3 or 4 children.

7. Let the children have a play party. Provide sets of cups and saucers, one for each child at the table.

* 8. Use a variety of procedures to have children determine whether sets match, and if not, which set has more members than the other. For example, give two children each a handful of blocks, cubes, beads, counters, etc. Have each child simultaneously remove objects, one at a time, until one child still has one or more objects remaining and the other has none. Use the term "more than" to describe what has happened.

   Ask the children to suggest other ways of determining which set has more members than the other.

9. To provide further experiences in comparing sets by one-to-one correspondence, present the following situations (or others that are applicable).

   a. Have someone bring a bag of peanuts to school. Ask the children for an easy way to tell, without counting, whether there are more children in the class or more peanuts in the bag.
   b. Without counting, decide whether there are more desks or more chairs in the room.
   c. Ask children how to determine whether there are more candle holders or more candles.

* 10. The following activity is suggested to illustrate
   a. that the things or persons in a set are the members of the set.
   b. that when the members of two sets can be put into one-to-one correspondence, they match.
   c. subset.

Call a group of 5 children to the front. Have the children suggest a name for this set, such as "Glucks", or "Set A", "the set Mary, Bob, Jim, Peter and Susie", "this set", "that set", etc. The name itself is unimportant.

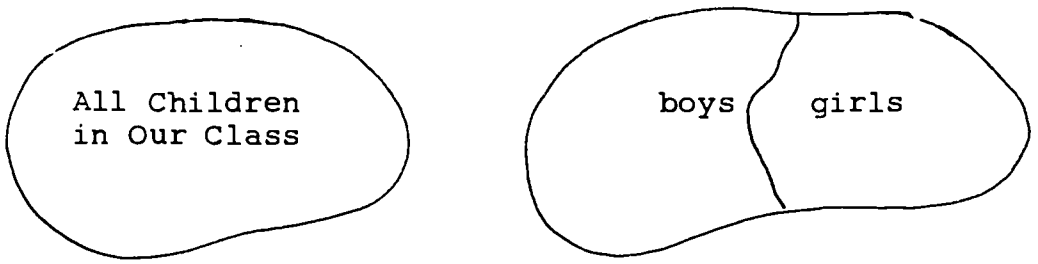
Call another group of 3 children to the front and determine a name for that set. Ask the children how they could tell which of the two sets has more members. When they suggest matching members of each set, ask them to think of different ways the matching could be done -- by pairs, joining hands, etc.

When the question of which set has more members has been answered, help the class to see that the two children who were "left over" form a new set. They may be given a different name. But note that they still are members of the original set, even while forming another set. This is the easiest subset to identify. Next, have the children look for other subsets that were formed by the above match. With each of the subsets discussed, point out that the members of the new set are also members of one of the first two sets.

11. Repeat the activity with sets of various sizes using a procedure similar to that suggested above. The activity may also be carried out with figures on a flannel or chalkboard.

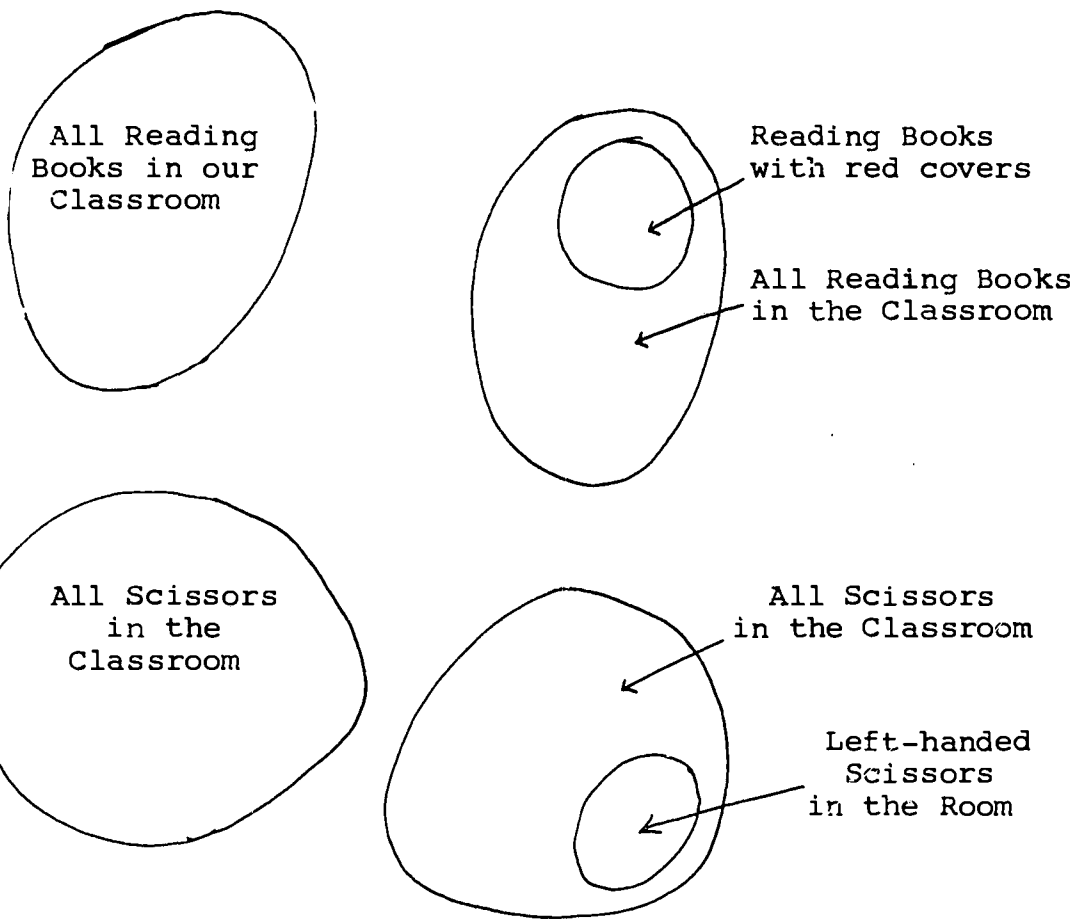
12. To develop understanding of subsets try this:

There can be sets that are part of other sets. The children in our classroom make a set.



Within that set we have a set of boys and girls. So we have two subsets whose members are also members of the one given set.

Other examples of subsets:



13. Ask the children if they can describe some of the subsets of David's toys (in the story "David's Promise").

14. Have all the girls in the room stand up. Indicate that this is the set of girls in the room.

    Have all girls wearing something red line up on one side of the room. This is a set of girls wearing something red. Ask the question, "Are all the girls lined up over here members of the set of girls in the room?" (Yes) They form a subset of the set of all girls in the classroom.

15. Say, "This class has another interesting subset, the set of all boys in this class. Will all members of this set raise their hands? This set is scattered all over the room. We can see it better if all the members come forward and stand in a row."

    "Is every member of this set a member of the class?" (Yes) "That is why we call it a <u>subset</u> of the class. The set of all boys in the class is a subset of the class. Each boy in the class is a member of the whole set, the whole class."

    "Tell me another interesting subset of the class." (Boys wearing blue jeans or any example given by one of the children.)

    "Let us check to see if this set is a subset of the class. Is every member of this set of boys wearing blue jeans a member of the class?" (Yes) "Then you are right! The set of boys wearing blue jeans <u>is</u> a subset of the class."

16. Have each child think of his own family. Have the children draw pictures of the members of their families and bring these pictures to the circle in the front of the room.

"These people form a set. What shall we name our set? Let's call it the set of "my family". Do you think there are any subsets in your sets? Let's try to think of all the subsets of our sets that we can. When I name a subset, I will ask a few of you which members of your set belong in that subset. Are you ready?"

Ask for the following subsets. After the children become acquainted with the idea, let them suggest other subsets.

(a) Subset of all children in your family
Danny, Kathy, Jimmy

(b) Subset of all adults in your family
Mom, Dad

(c) Subset of all boys in your family
Danny, Jimmy

(d) Subset of all girls in your family
Kathy

(e) Subset of all people living in your family
Mom, Dad, Danny, Kathy, Jimmy

(f) Subset of all members with black hair in your family
Dad, Danny, Jimmy

When the subset of all of the people living in their house is asked for it may be necessary to explain that even though this will include all of the members of the original set, it can still be called a subset. A set is a subset of itself. You might ask the class to

suggest other instances in which the subset will be the same as the original set. Other examples are all the members of "my family" who wear clothes, who sleep, who laugh, etc.

* 17. Show the class a set of 5 objects (6 if the class has more than 32 members) which are easily distinguished and drawn. Give each child a blank sheet of paper and ask him to draw a subset of the set displayed. Each child should be allowed to show his subset if it is different from everyone elses. If two children picture the same subset (the same members), the second should be given another sheet and asked to draw a different subset. Since there are 32 different subsets (counting the empty set and the entire set), each child can draw a subset different from everyone elses.

Commentary on Worksheets 4, 5, and 6 - Subset

These worksheets are designed to give the children experience with subset. Here, as elsewhere, the worksheets should be preceded by similar activities, which may make some of the worksheets unnecessary. The starred worksheets should be used anyway, to aid in the evaluation of the program.

Worksheet 4

In activities with common objects such as those depicted on Worksheet 4, it is preferable for the teacher to use real examples of the objects. In other words, in this case, real boxes of gelatin could be placed on a table in front of the group.

Start the discussion by asking the children to notice the differences in the boxes of gelatin. Differences in the fruit pictured on the box are usually pointed out, although some child may comment on the fact that the words look different. Accept this response also.

Suggest ways of identifying the subsets. One way would be to have them draw a yellow mark on each box that has a lemon on it; a red mark on the boxes with strawberries; and a blue mark on the boxes with raspberries.

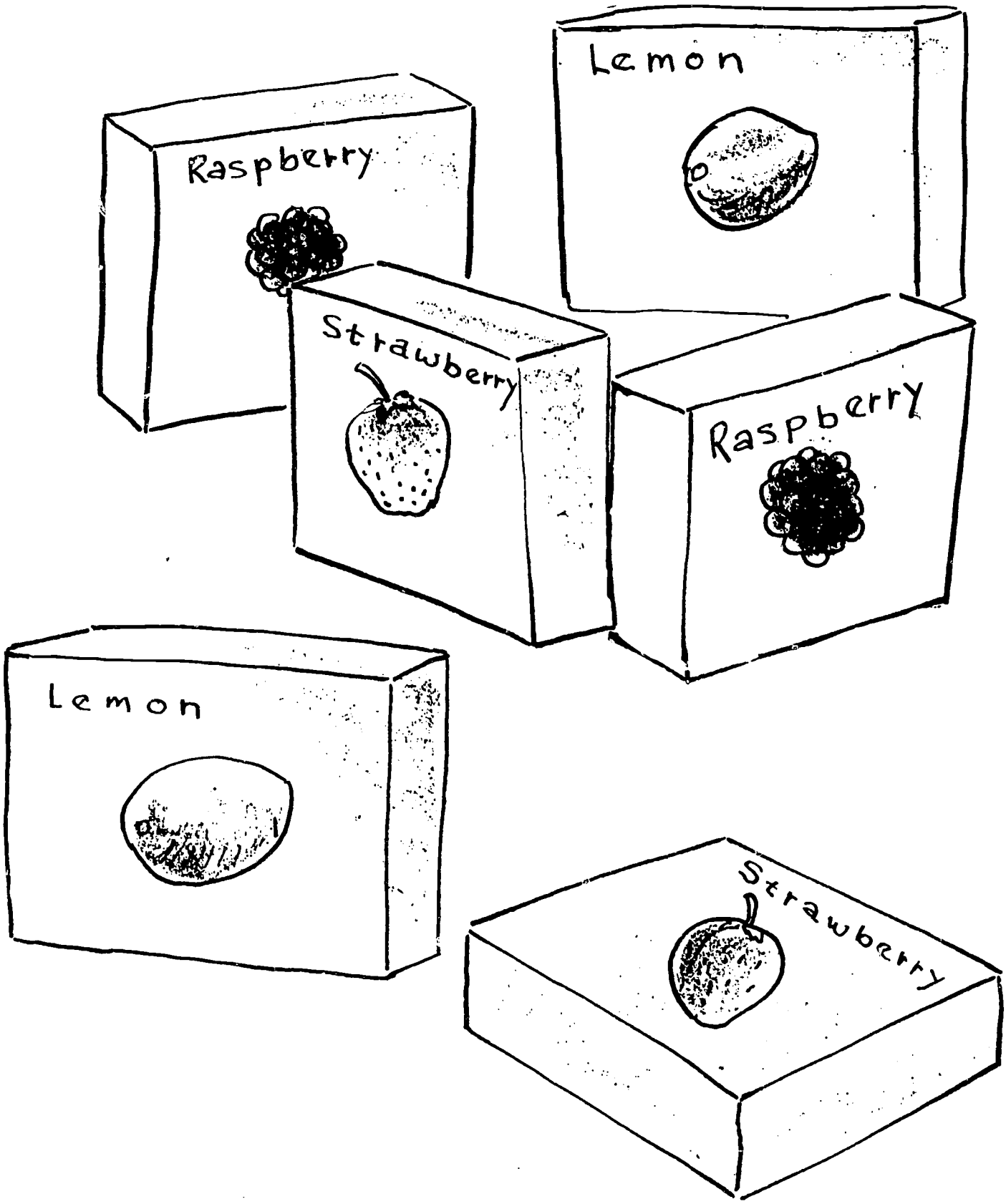
Worksheet 5

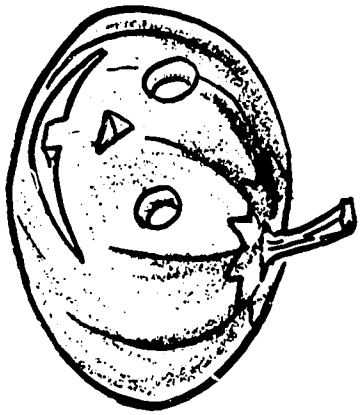
Point out that there are many ways of classifying or grouping objects. In row 1, for instance, if we consider the subset of things we do when it is cold, then the boy swimming is the only one not a member of the subset. If we consider the subset of things made of snow, then the snowman is the only member. Some child may consider the subset of summertime activities. The boy swimming is the only member of this subset.

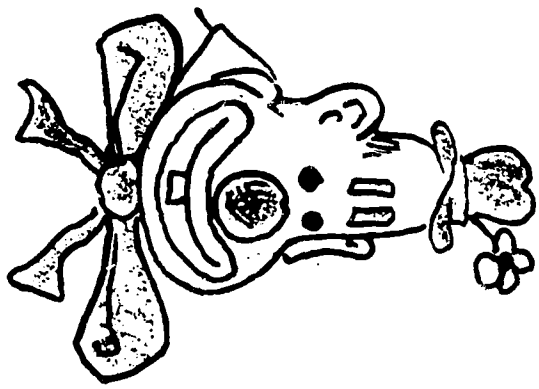
Point out and discuss the various subsets in each row. Have the children explain the reason for their choices. Have the children put an X on the members of a subset in each row. Since any combination of members is acceptable under the definition of subset, all answers are acceptable, even a subset of one member, the empty set, or the whole set.

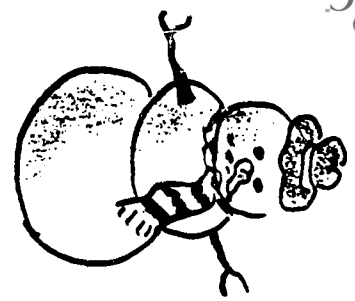
* Worksheet 6

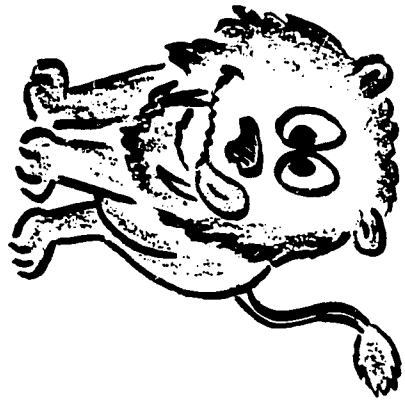
Have the children put an X on each member of the subset of cookies they'd like to eat. When they've done this, tell them to draw a simple closed curve around the subset of cookies they would not like to eat. If they have already indicated they'd like to eat all of the cookies pictured, have them circle an empty space. This is an excellent illustration of the mathematical fact that every set has at least two subsets - the original set itself and the empty set.

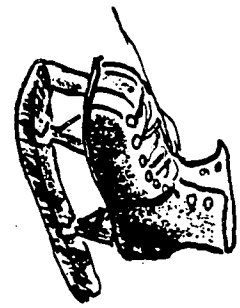

Lemon
Raspberry
Strawberry
Raspberry
Lemon
Strawberry

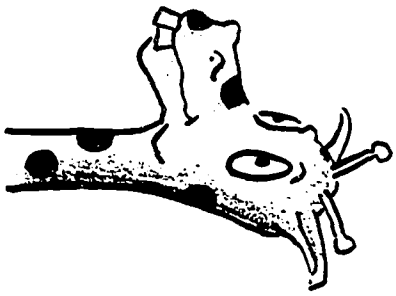


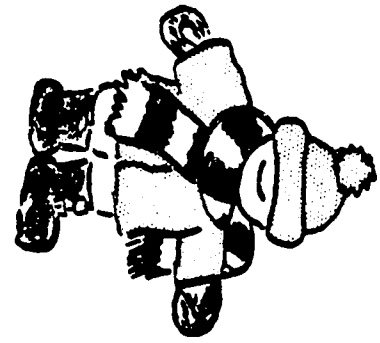


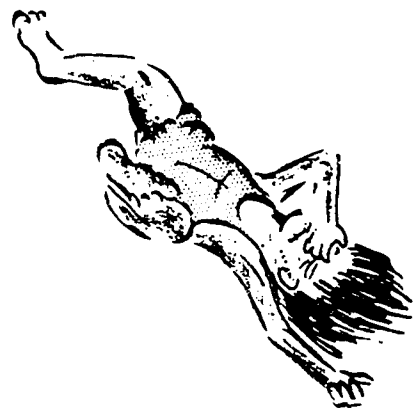


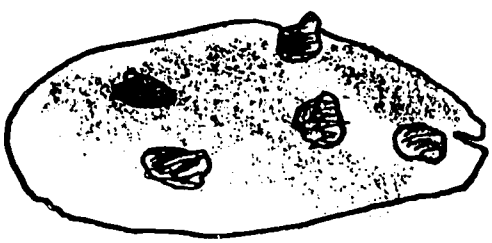


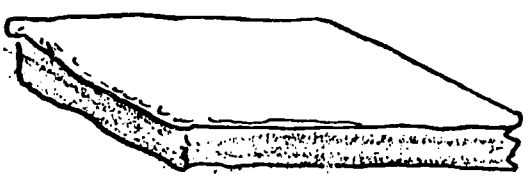


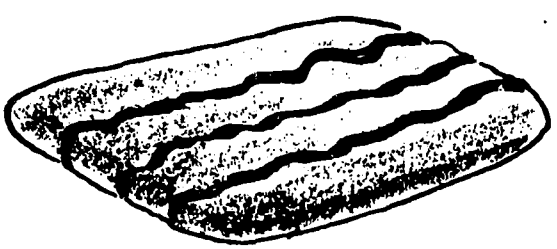






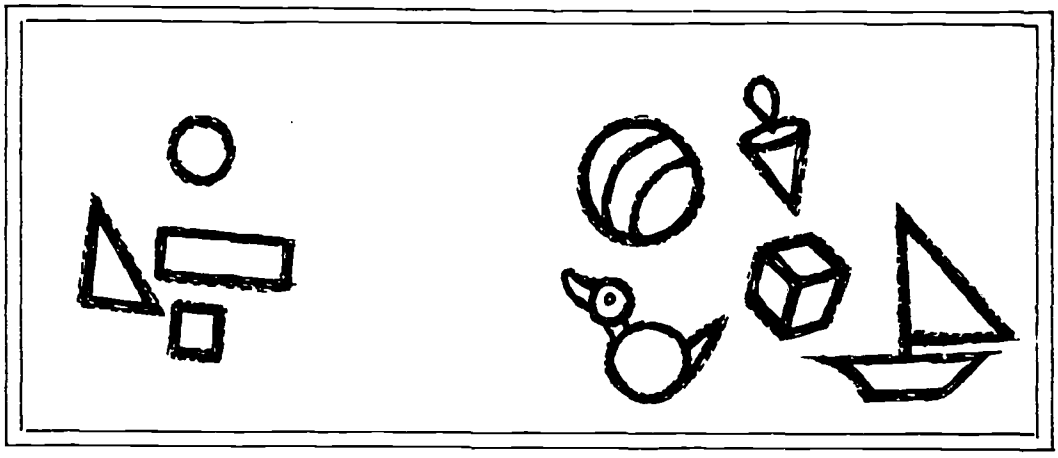






* 18. Use the flannelboard for the following activity. Place two sets of flannelboard cut-outs on the board. The example suggested is a set of 5 pictures of toys and a set of 4 geometric shapes (smaller than the pictures of the toys).

Arrange the set of geometric shapes so they occupy less board space than the set of toys. Ask which set has more members than the other set. (toys)



Move the geometric shapes further apart from each other, so they occupy more space than the set of toys. Ask if moving the objects made any difference as to which set has more members than the other. Elicit the response that there is no difference because we still have the same set.

Next, substitute another, larger object for one member of the set of geometric cut-outs. Ask if we still have the same set as before. (No) Then pose questions to bring out the fact that the set of geometric shapes still has less members than the other set.

Repeat the above activity until the children understand that the substitution of members of a set or the re-arrangement of the members does not affect the one-to-one correspondence between sets.

19. This activity will give the children experience in estimating and matching for one-to-one correspondence. A supply of seashells, stones, beads or other small objects is needed.

Choose two children to play at one time. Have one child take a handful of beads, the other a handful of shells.

Have each child place his set on the table in front of him. Ask that the child who thinks he has more in his pile raise his hand. Test this by having the children match objects from their pile, one by one, until one of the piles is empty. The child who estimated correctly is the winner. Be sure to emphasize that the unmatched members form a subset of the larger set.

20. Draw a picture of an egg carton and a set of eggs on the chalkboard. Draw a line from one egg to one empty compartment in the carton. Have the children go to the board, one at a time, and draw a line from one egg to a matching empty space in the carton. Then ask:

"Are there any spaces without eggs?"

"Are there any eggs without spaces?"

Have the children take a piece of colored chalk and draw a simple closed curve around the eggs that are left. (Don't count them.) Then ask them if they can tell, without counting, which set has more members - the set of eggs or the set of spaces in the carton.

Someone might suggest matching the eggs and spaces by erasing one egg from the set of eggs and drawing it in an empty space in the carton. This is permissible, since it is another way of indicating one-to-one correspondence. If a more concrete approach is desired, use actual egg cartons and plastic eggs (or large beads).

21. Post two sets of figures on the flannelboard. Put more members in one set than in the other. Ask a child to go to the board and point to the set that has more members. Provide similar experiences, using the chalkboard, newspaper pictures, etc.

22. Draw a pair of sets on the chalkboard, one set having more members than the other. Ask a child to put an X on a member of the set which has more members. After a few times, let a child draw the sets and choose a classmate to mark them. Continue this way until all children have an opportunity to either construct or mark the sets.

    If someone puts the same number of members in both sets, the "marking" child might use lines to connect the members of one set with the members of the other to show that the sets match.

Commentary on Worksheets 7-13

Worksheet 7

Ask the children:

"Which set has more members than the other?"

"How can we test (or prove) what you say?"

Have them show that we can match the sets by drawing lines from a member of one set to a member of the other set. The sets match if all of the members of both sets are used up or connected. One set has more members than the other if it has members that are left over after the matching process.

Worksheet 8

Worksheets 8a and 8b are used together; 8a is a discussion sheet, 8b the activity sheet. The same two sets of figures are presented three times on each sheet. The purpose of the worksheet is to show that while there are many different ways of comparing sets by one-to-one correspondence, the result is always the same. In this case, there is always one object of the right hand set left over.

Before using Worksheet 8, it may be helpful to do similar exercises on the flannelboard or chalkboard. After one way of connecting the members is shown, a child may illustrate another way by changing some of the connecting lines or using different colored chalk.

On Worksheet 8a discuss the differences in the ways the members are connected (or paired or joined). On Worksheet 8b the children are asked to connect the members in three different ways which are also different from those shown on 8a. Altogether there are 24 different ways these sets can be compared. After the children complete 8b, discuss in what way all of their pictures are alike (they all have an unmatched toy).

Worksheet 9

Discuss the fact that there is a set of large balls and a set of small balls. Ask the children to estimate whether there are more large balls than small balls. To check their prediction, have them draw a line and connect one large ball with one small ball until all the members of one set have been used up. Ask them to put an X on a member of the set which has more members than the other set.

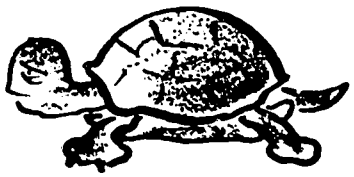
* Worksheet 10

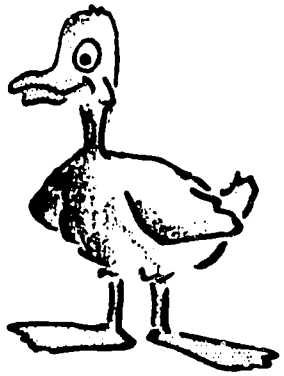
This worksheet has been designed to discourage counting. Ask the children if the sets match. If they say no, have them put an X on a marble of the set which has more members. Have them prove their estimates by drawing lines to connect members of each set.

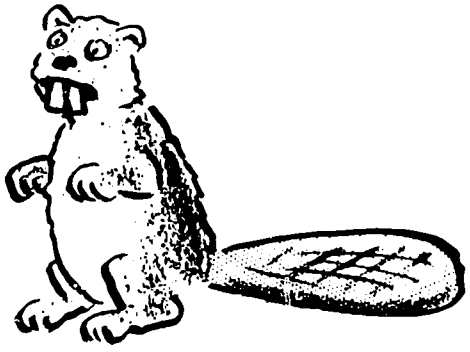
* Worksheet 11

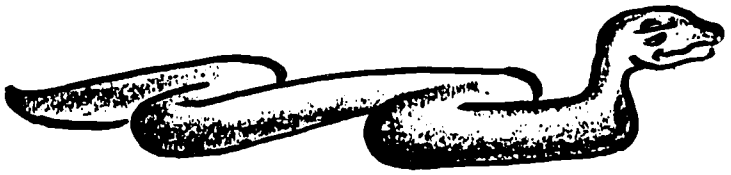
. Present in same way as Worksheet 10.

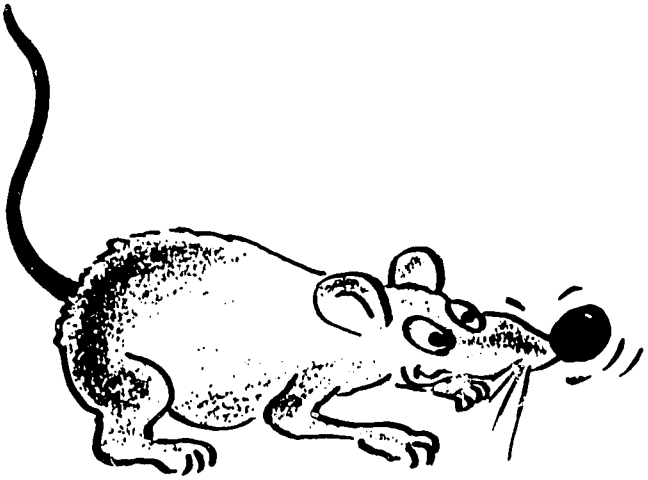
* Worksheet 12 is self-explanatory.

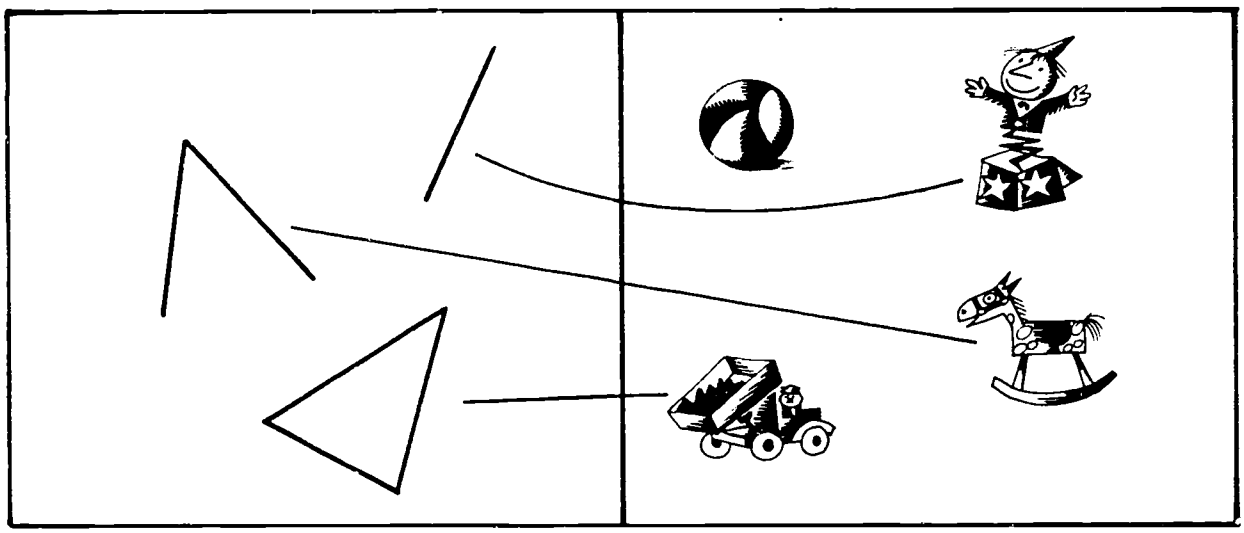


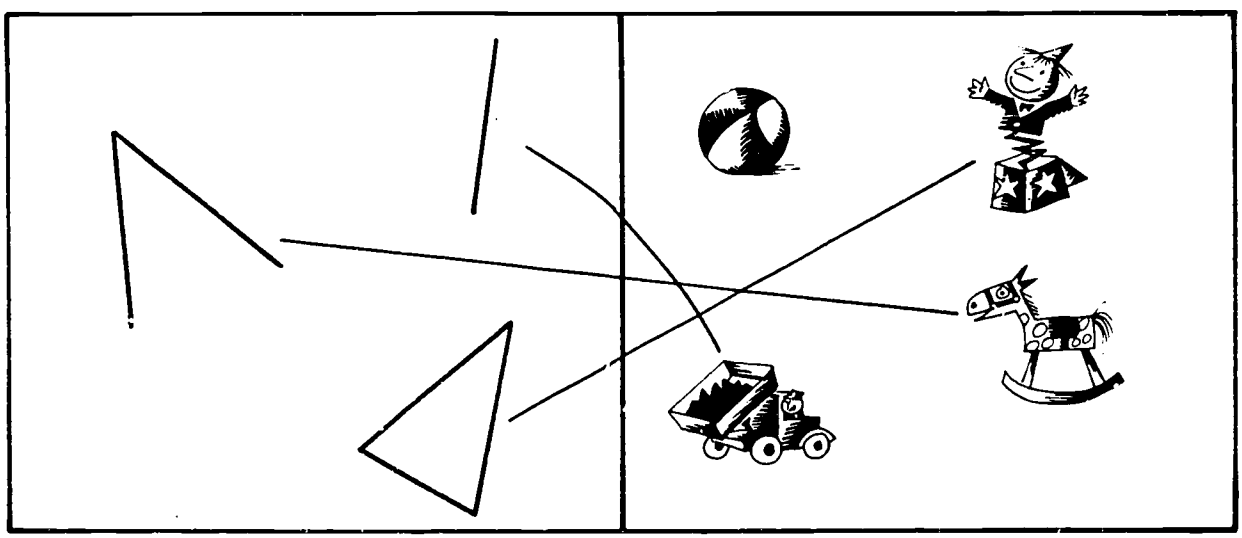


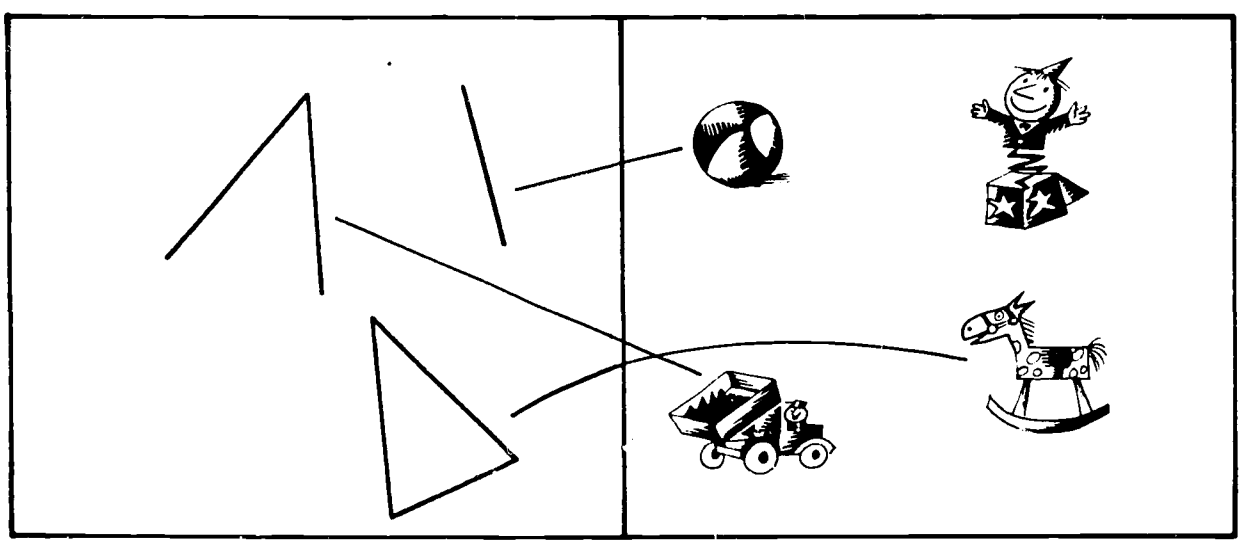




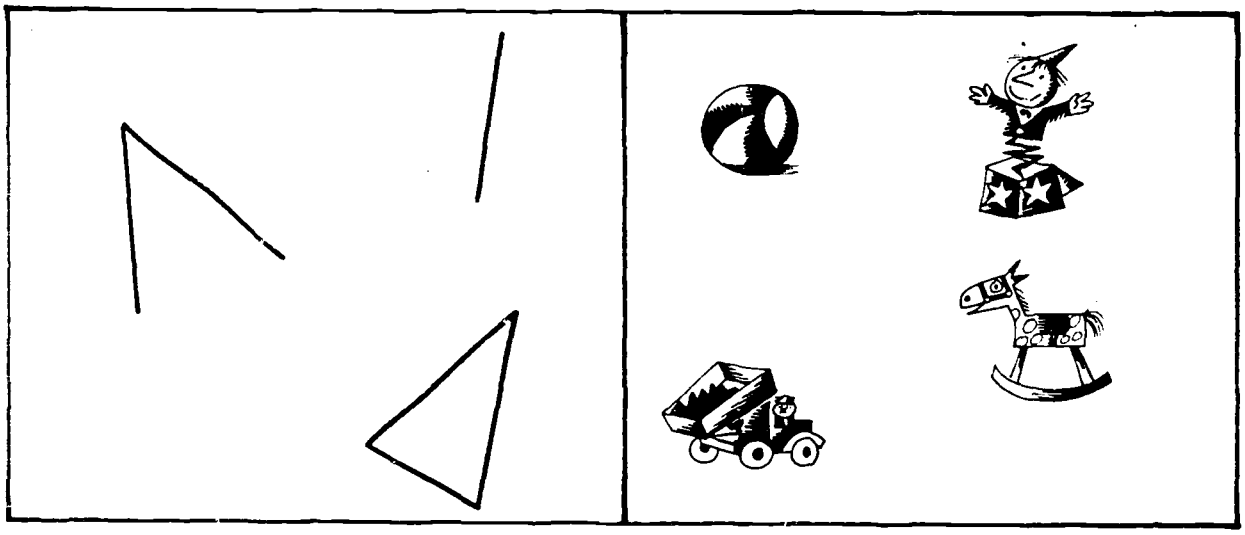


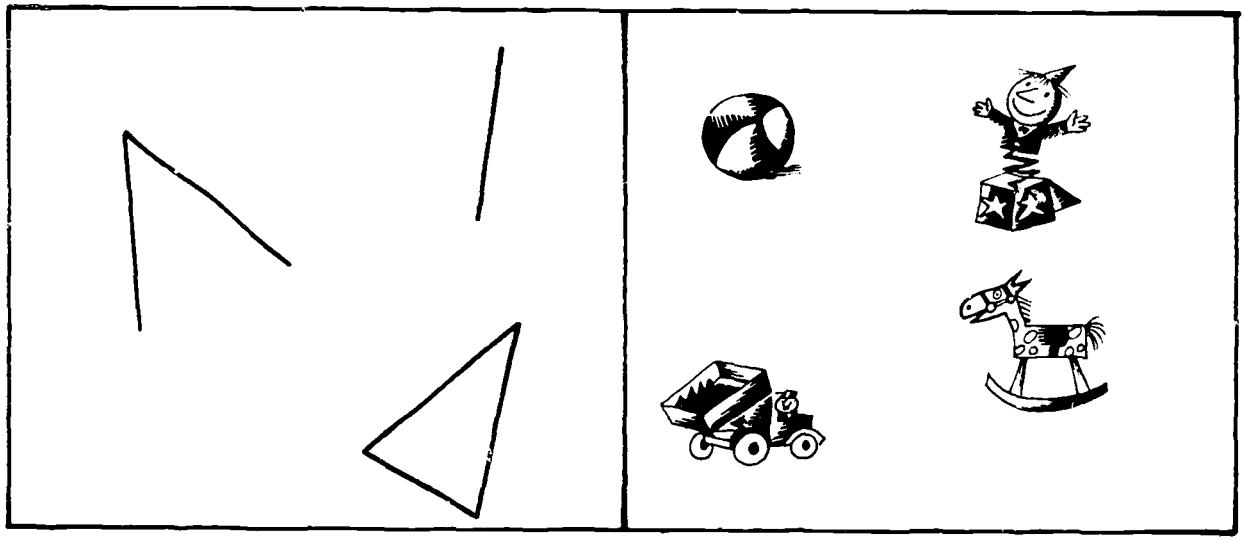


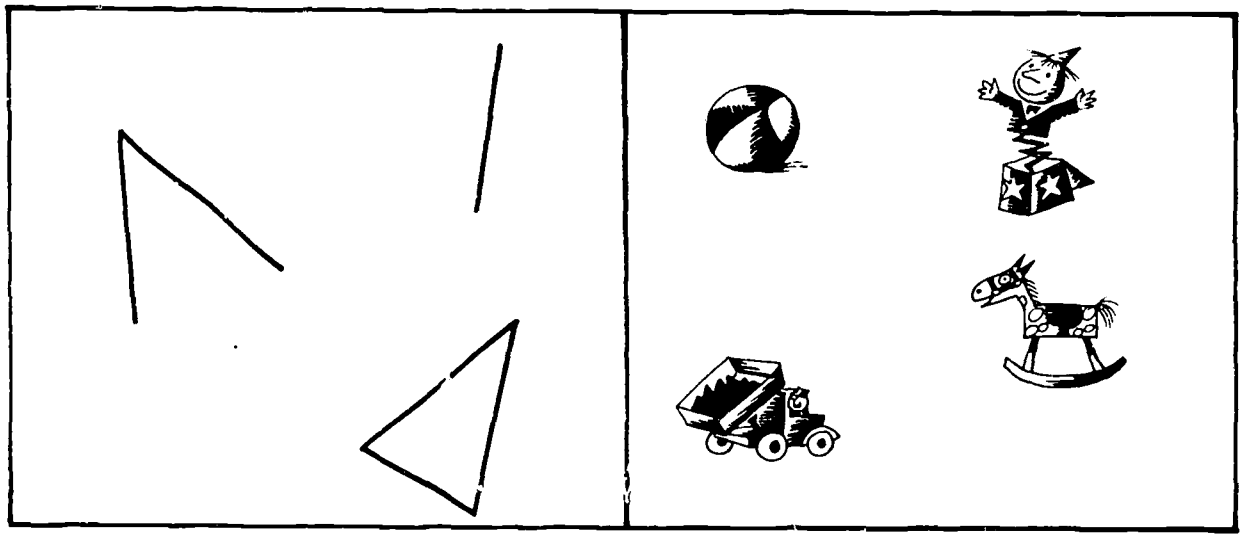


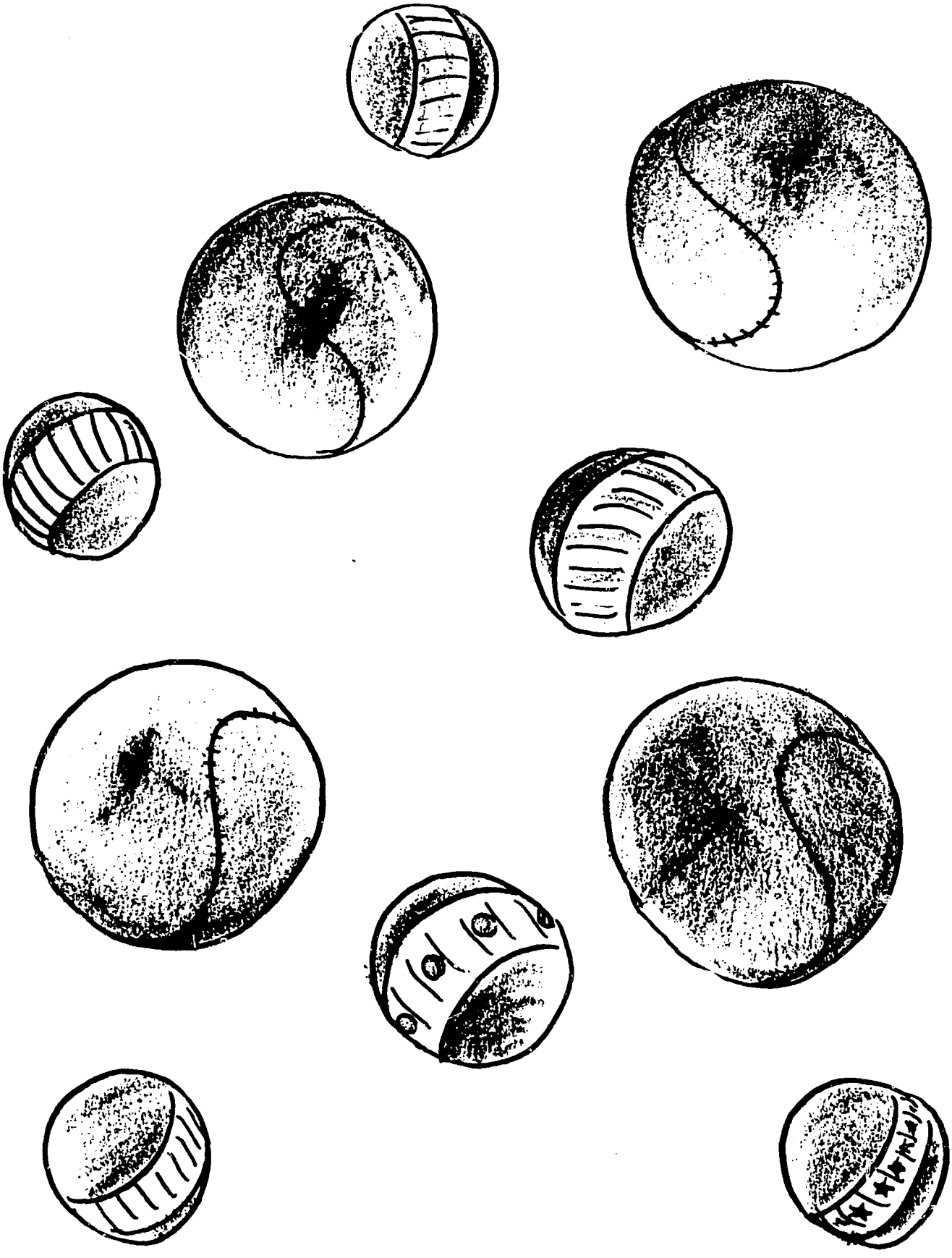


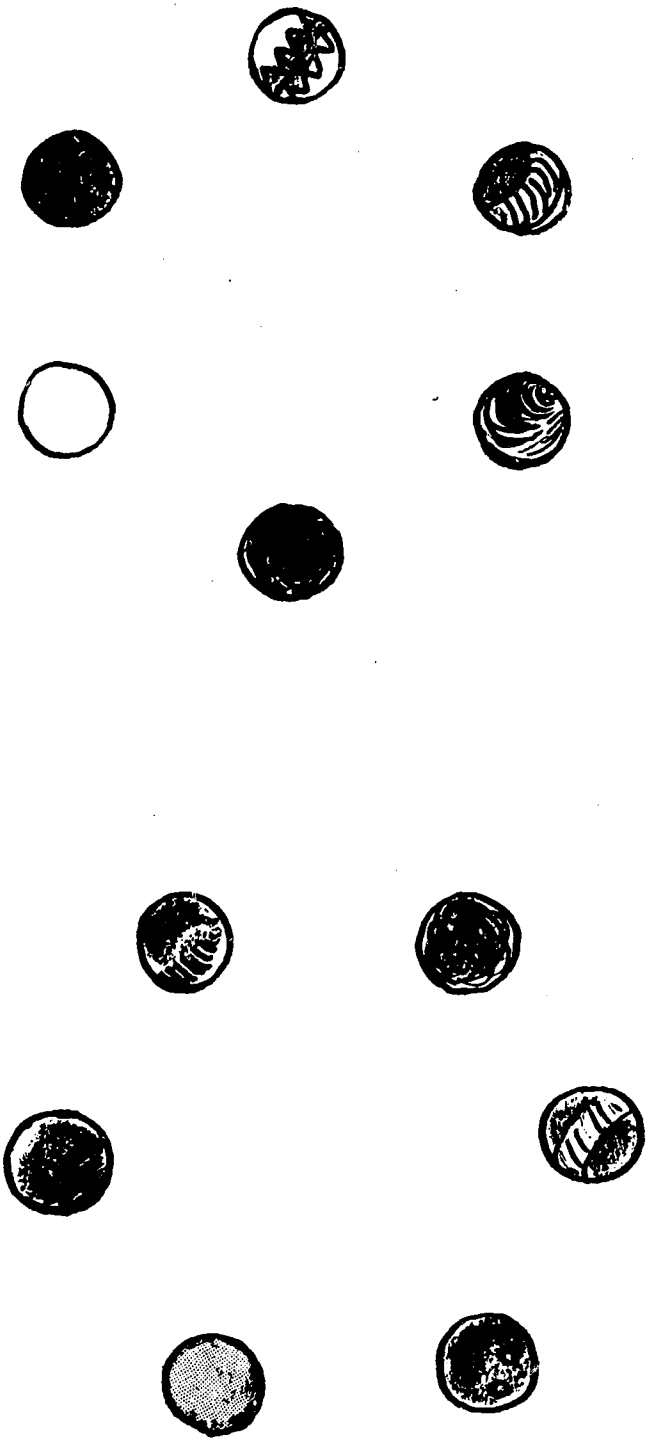
One-to-One Correspondence











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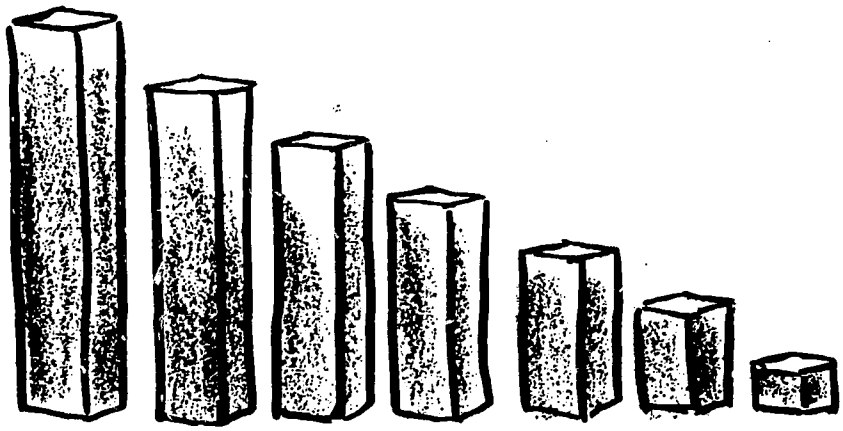

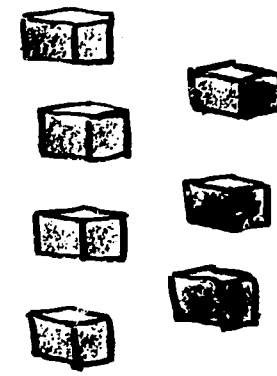

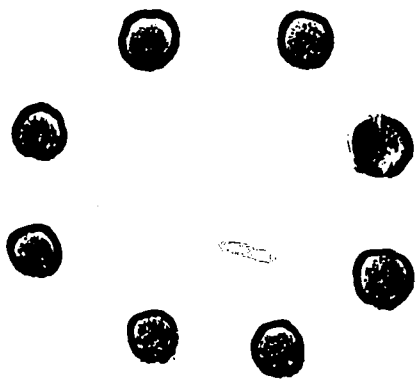

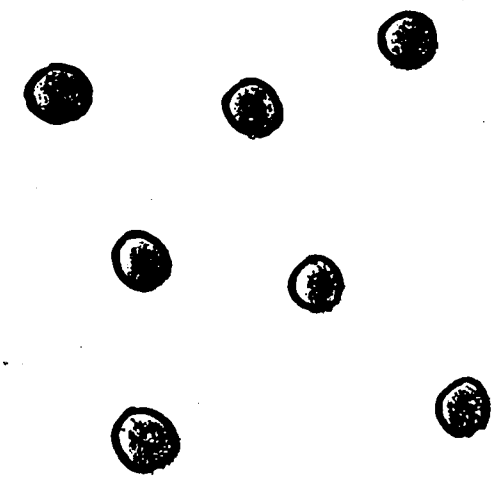

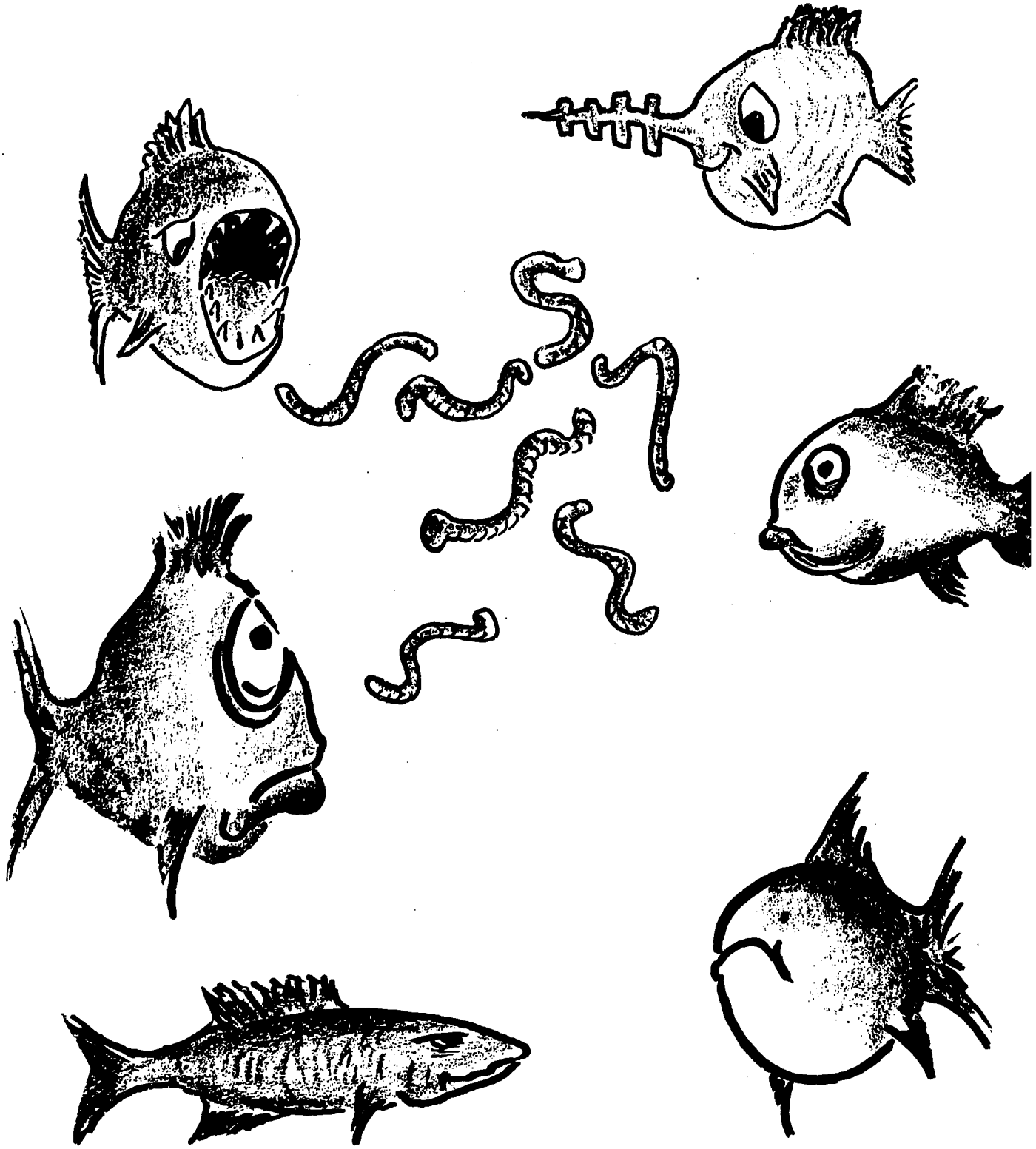

Here are some fish and some worms.

Make an X on a member of the set which has more members.

* NIM

Nim is a strategy game. In order to play it successfully, it is necessary to discover at least one of the many strategies contained in the game. Most children will probably be able to deduce some working rules which will help them win. Different strategies will be discovered at different levels of maturity.

The only test for the strategy discovered is the question, "Does it work?" There is one best strategy for this game.

Discovering the game strategy is not an isolated process. It is basic to other mathematical concepts which will be introduced later on in the Minnemast units. However, it is game strategy and ought to be used in a situation resembling the playing of a game. The class may become generally fascinated with the challenge of discovering the "winning way". If the entire class does not take to it, the teacher ought to watch for individuals who show particular curiosity and encourage them. The game should not be required of those who lack interest in it. Other strategy games will be included in later Minnemast materials. Spontaneous interest will probably appear when the children acquire a greater degree of maturity. The enthusiasm of the other children will soon be whetted. Let those who know the "trick" move around and play with the others who don't know it yet.

The game may be played as follows:

Two children play the game at a time. An equivalent set of any number of red checkers and a set of any number of black checkers are placed before them in two piles. Each child, in turn, may take any number of checkers from either one of

the sets (this would be a subset of the set of checkers). In fact, he may take a whole set if he wishes. He must, however, take at least one checker. A player may not take from both sets at the same time. The game continues until all the checkers from both sets have been taken. The child who takes the last checker wins the game. The game may be played with sets of any two kinds of objects - blocks, dolls, hats, boys and girls, etc.

If a player always leaves the same number of checkers in each set, he is sure to win. Do not tell this to the children, however. Tell the children that there is a winning strategy, and that when they find it they should keep it a secret. Encourage those who have discovered the strategy to test their discovery by trying it with different partners.

For added interest, give the child who discovers the strategy a special card labeled NIM which he may decorate and display on his desk either right-side up, so it reads NIM or turned, to read WIN.